# A Partial Mastery Grading Approach for Calculus

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Spring 2023 MD-DC-VA Section Meeting
Virginia State University

# Warm-Up

## Think, Pair, Share

- How might you define a “typical” or “traditional” grading system?
- What grading system do you usually use (in Calculus specifically, if applicable)? Why?
- What is your interest in this session?

# Stevenson University


- Private university in Baltimore County, MD
- Approximately 2700 undergraduates
- Precalculus and Calculus primarily taken by students majoring in Applied Math, Chemistry, Biochemistry, and Biomedical Engineering and minoring in Math
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- Home of the Fall 2023 MD-DC-VA Section Meeting!!!! Oct. 13–14, 2023

# Textbooks

**Active Calculus: https://activecalculus.org/**

Active Prelude to Calculus, Active Calculus, and Active Calculus Multivariable are free, open-source texts designed for a more active learning experience for students. Every section of each text has engaging activities for students to complete before and during class, as well as exercises that challenge students to connect and assimilate core concepts.


# Partial Mastery Grading

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In mastery grading, student work is graded directly on whether it meets a clear list of outcomes. Rather than awarding points or partial credit, clear expectations are set in advance for how student work will be assessed, and the instructor evaluates whether or not these expectations have been met.

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## Partial Mastery Grading

Not all portions of a student's grade is graded by mastery.

# Course Requirements

| Component | Percentage |
|---|---|
| Online Homework | 18% |
| Core Learning Targets | 26% |
| Exam 1–3 | 39% (13% each) |
| Final Exam | 17% |

## Core Learning Targets

There are 13 core learning targets (CLTs) in the course. CLTs are graded on a mastery-based scale: each target receives either "mastered" (M) or "not yet" (NY). Students have 2–4 attempts at each of the 13 CLTs on quizzes and exams and 1 additional attempt to master each CLT outside of class. Each mastered CLT contributes 2% to the semester grade; if they master all 13, they will earn the full 26%.

# Precalculus Core Learning Targets

1. I can solve linear equations and basic nonlinear equations.
2. I can use function notation, find and simplify the output of a function given an input, and find an input that produces a given output. I can interpret the meaning of a function in applied contexts. I can do these for functions given by a formula, a table, and a graph.
3. I can find the average rate of change in a function on a given interval and state the units and interpret the meaning of the average rate of change in applied contexts.
4. I can find an equation for any linear function in point-slope and slope-intercept form, given appropriate data or information. I can interpret the meaning of a linear function and its components (slope, intercept(s), etc.) in applied contexts.
5. I can find the $x$-intercepts, $y$-intercept, and vertex of a quadratic function. I can do this for a quadratic function given in standard form, vertex form, and factored form.
6. I can find and simplify values of the composition of two functions.

# Precalculus Core Learning Targets

7. I can determine whether a function has an inverse, find values of the inverse given input-output pairs of the original, and find the formula of the inverse of basic functions that have an inverse. I can interpret the meaning of an inverse function including the units of its input and output.
8. I can find the amplitude, period, and midline of a transformed version of the basic sine or cosine function, and I can find the formulas for transformed versions of the basic sine or cosine function that have certain described properties.
9. Given two points or an appropriate collection of features, I can find a formula for an exponential function that has those characteristics.
10. I can find exact solutions to equations involving exponential and logarithmic expressions.

# Precalculus Core Learning Targets

11. I can identify the intervals where a function is increasing, decreasing, concave down, or concave up, and I can provide examples of familiar functions that have given increasing/decreasing and concave down / concave up behaviors.
12. Given a right triangle with partial information about its sides and angles, I can determine the missing information for the remaining sides and angles, even if the quantities involve an unknown variable.
13. I can identify the degree, zeros, turning points, and long-range behavior of a polynomial given its formula or graph, and I can find the formula of a polynomial that fits given data about its behavior.

# Calculus Core Learning Targets

1. I can find the average velocity of a moving object or a more general average rate change of a function on a given interval, state the units, and interpret the meaning.
2. I can find limits algebraically, graphically, and numerically.
3. I can find the derivative of a function at a point or the derivative function of a function using the limit definition.
4. I can interpret the derivative as the slope of the tangent line to a curve at a point and find the equation of a tangent line. I can interpret the meaning of the derivative as an instantaneous rate of change and find the units of a derivative in applied contexts.
5. I can find a derivative using elementary derivative rules for constant, power, exponential, sine, and cosine functions, together with the Constant Multiple and Sum Rules. I can use these derivatives to solve problems.

# Calculus Core Learning Targets

6. I can apply the product and quotient rules to find derivatives in conjunction with previous derivative rules. I can use these derivatives to solve problems.
7. I can apply the chain rule to find derivatives in conjunction with previous derivative rules. I can use these derivatives to solve problems.
8. I can find derivatives of functions involving the other trigonometric functions (tangent, cotangent, secant, and cosecant) and inverse functions (logarithmic and inverse trigonometric functions) in conjunction with previous derivative rules. I can use these derivatives to solve problems.
9. I can use derivatives in at least one of the following applications:
    - Evaluating limits using l'Hôpital's rule
    - Identifying absolute minimum and maximum values of a continuous function on a closed interval
    - Applied optimization problems

# Calculus Core Learning Targets

10. Given appropriate information about a function, I can find its critical numbers, construct a first derivative sign chart, identify where a function is increasing and decreasing, and determine where a function has local extrema.
11. Given appropriate information about a function, I can construct a second derivative sign chart, identify where a function is concave up and concave down, and determine where a function has points of inflection.
12. I can use a Riemann sum to estimate the area between a given curve and the horizontal axis over a particular interval. I know the differences among left, right, middle, and general Riemann sums. I understand the connection between Riemann sums and the definite integral.
13. I can find an antiderivative of a basic function using my knowledge of derivatives. I can find the exact value of a definite integral by applying the Fundamental Theorem of Calculus.

# Schedule

| Quiz/Exam | CLTs |
|---|---|
| Quiz 1 | 1 |
| Quiz 2 | 1, 2 |
| Quiz 3 | 2, 3 |
| Quiz 4 | 3, 4 |
| Exam 1 | 1–4 |
| Quiz 5 | 4, 5 |
| Quiz 6 | 5, 6 |

| Quiz/Exam | CLTs |
|---|---|
| Exam 2 | 5–8 |
| Quiz 7 | 7, 8 |
| Quiz 8 | 8, 9 |
| Quiz 9 | 10, 11 |
| Exam 3 | 9–11 |
| Quiz 10 | 12, 13 |
| Final Exam | 1–13 |

4. I can interpret the derivative as the slope of the tangent line to a curve at a point and find the equation of a tangent line. I can interpret the meaning of the derivative as an instantaneous rate of change and find the units of a derivative in applied contexts.

   a) Consider the function $f(x) = 2\sqrt{x+3} + x$. Use the fact that $f'(1) = 1.5$ to find an equation of the tangent line to $f$ at $x = 1$.

   b) Let $V(t)$ represent the volume (in cubic centimeters) of water in a bucket $t$ seconds after a gardener begins filling it.

      i) Write a sentence to explain the meaning of the expression $V(10) = 2000$. Make sure to include units in your explanation.

      ii) Write a sentence to explain the meaning of the expression $V'(30) = 450$. Make sure to include units in your explanation.

5. I can find a derivative using elementary derivative rules for constant, power, exponential, sine, and cosine functions, together with the Constant Multiple and Sum Rules. I can use these derivatives to solve problems.

   a) Find the derivative of each function using elementary derivative rules (for constant, power, exponential, sine, and cosine functions, together with the Constant Multiple and Sum Rules).

      i) $f(x) = 3\sqrt{x} + \dfrac{4}{x} + 2(5^x)$

      ii) $g(x) = \dfrac{1}{x^2} + e^x + 3x$

   b) The height (in feet) of the peak of a wave in a wave pool $t$ seconds after you begin measuring the waves can be modeled by the function $h(t) = 2\sin(t) + 4$.

      i) Find a formula for $h'(t)$.

      ii) Compute $h'\left(\frac{11\pi}{6}\right)$. Write a sentence to explain the meaning of the expression $h'\left(\frac{11\pi}{6}\right)$. Make sure to include units in your explanation.

# Sample Questions

6. I can apply the product and quotient rules to find derivatives in conjunction with previous derivative rules. I can use these derivatives to solve problems.

   a) Find the derivative of each function the product and quotient rules to find derivatives in conjunction with previous derivative rules.

      i) $r(x) = x^3 \sin(x)$

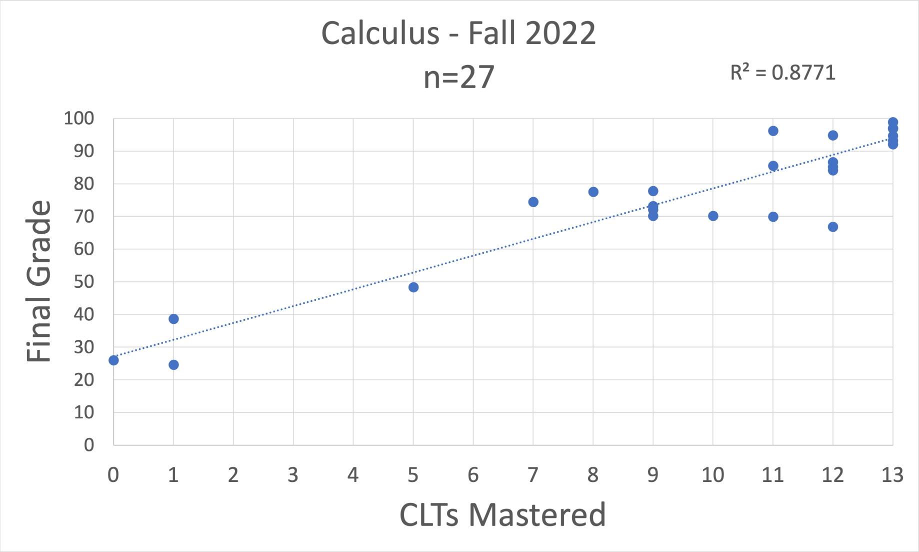
      ii) $s(x) = \dfrac{4^x + 5}{x^3 \cos(x)}$

   b) Consider the function

$$h(x) = 3xe^x + \frac{x^2 + 2}{\sin(x) + 1}.$$

   Find an equation for the tangent line to $h$ through the point where $x = 0$.

# Data


Calculus - Fall 2022
n=27
$R^2$ = 0.8771
100
90
80
70
60
50
40
30
20
10
0
Final Grade
0
1
2
3
4
5
6
7
8
9
10
11
12
13
CLTs Mastered

# Benefits and Pitfalls

## Benefits

- Encourages students to come to office hours
- Clear learning goals from the outset
- Students that put in the effort usually do well
- Students can try again until they master a concept
- Helps remove some of the anxiety students have about taking tests and quizzes in math
- Easier to grade

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## Pitfalls

- No partial credit
- Students that want to "coast" to a C may have trouble
- Grades can swing drastically in Blackboard
- More confusing for students than standard grading

Thank you!

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