Mathematical Association of America
MD-DC-VA Section, April 12-13, 2013
Salisbury University – Salisbury, Maryland
Abstracts

Abstracts for the workshop and invited addresses are listed first, in chronological order, followed by faculty and graduate student abstracts, alphabetized by first presenter's last name. Student presentation abstracts follow, with student poster abstracts at the end (all alphabetized the same way).

Invited Addresses

FRIDAY WORKSHOP

Jean McGivney-Burelle, University of Hartford
Ann Stewart, Hood College
Teaching with Classroom Voting and Clickers
4:00 PM, Henson Hall, Room 150

This minicourse will provide participants with an overview of classroom voting pedagogy in a wide range of college mathematics courses. We will discuss the logistics of classroom voting as well as recent research on this type of pedagogy. We will also share our thoughts on writing effective voting questions. Participants will play the role of students in a voting demonstration, explore an online library of over 2300 classroom voting questions, and prepare a lesson with voting questions for use in one of their own courses.

BANQUET ADDRESS

Jason Rosenhouse, James Madison University
Knights, Knaves, Normals, and My Nephew
8:00 PM, The Bistro, Salisbury University Commons

Puzzles about liars and truthtellers have a long history, but they were elevated to a high art by Raymond Smullyan. In books like *What is the Name of This Book?, The Lady and the Tiger* and *Forever Undecided*, Smullyan shows how elementary logic problems can lead the reader gradually towards sophisticated ideas like Godel's famous theorems. I first encountered Smullyan's writing in elementary school, and his books have had a major influence on me. I was recently invited to edit a tribute volume in his honor, and my work on this project afforded a welcome opportunity to reread his numerous books. In this talk we shall discuss some elementary features of these sorts of puzzles, develop a basic theory for thinking about them, and eventually build up their use for explaining difficult ideas in logic. Most of the talk will be readily accessible to students.

SATURDAY INVITED ADDRESSES

John Hamman, Montgomery College
Proving the Impossible
9:55 AM, Purdue Hall, Room 156

From some of the earliest proofs in mathematics to some of the most important modern results, mathematicians have sought to demonstrate that some things cannot be done. We will consider such proofs from pre-Euclid to post-Gödel, with a few stops in between. However, instead of simply focusing on the results themselves, we will consider the language used in these proofs. The way we choose to describe mathematics can have a profound impact on our relationship to the content and can affect how non-mathematicians view it. This is particularly true when the result in question is a proof of impossibility.

Robert Allen, University of Wisconsin, La Crosse
Surviving an Outbreak of Zombiism
3:35 PM, Purdue Hall, Room 156

Since the 1930's zombies have played an important role in the American horror film genre. Have you ever wondered if a zombie apocalypse is realistic and/or inevitable? Can we use mathematics to help us fight off such an apocalypse? In this talk I will discuss a series of models for the spread of a zombie infection through a human population. These models are based on the standard SIR model, modified to introduce various coping techniques as well as dynamical behavior. Although an outbreak of zombiism is “fictional”, I will discuss how mathematics is used to study the spread of an infectious disease, and the work can be translated to studying “real” diseases such as cholera, chicken pox, H1N1, and HIV, for example.

Contributed Faculty Papers by Author

Abdinur Ali, Norfolk State University
Mushtaq Khan, Norfolk State University
Computational Modeling of Photonic Crystal Band Gap Fibers
9:25 AM, Henson Hall, Room 113
Regular optic fibers are made up of two glass fibers. The inner glass is the core and it has a high refractive index. This core is placed inside of the outer glass. The outer glass is the cladding and it has a low refractive index. The optical waves are guided in the core fiber using total internal reflection techniques. However, for photonic crystal band gap fibers, the fiber is made up of hollow core in silica surrounded by lattice of silica-air holes acting as cladding. The concept of the total internal reflection does not apply in this case. The only way to keep the optical waves in the core is to use band gap structures. These band gap structures confine the light in the core and preventing it from spreading. Some of the design parameters for the band gap structures are the ratio of the diameter of the air holes and the periodicity of the silica-air holes. This paper explains the difference between the design of conventional fibers and hollow-core band gap fibers. It also covers the computational modeling of the design parameters for the hollow core fibers such as periodicity, periodic cladding and the design of the single mode fibers.

Jathan Austin, Salisbury University
Further Explorations in Counting and Divisibility for Undergraduate Mathematics
2:15 PM, Henson Hall, Room 111
I will share a number of tasks in which students must count collections of integers with given divisibility properties or construct an integer with given divisibility properties. I will discuss potential uses of these tasks when teaching college mathematics. In particular, the tasks are applicable to the teaching of topics in discrete mathematics and elementary number theory.

Tauqir Bibi, South University
Using Online Discussion Forums to Enhance Student Engagement
9:00 AM, Henson Hall, Room 107
Most South University students are working adults trying to enhance their credentials by going to school. Day classes meet twice a week, and the night classes meet once a week. Due to the large class size, and insufficient time, several students do not get the opportunities to ask questions during the class. In addition, some students want to ask questions about their assignments on the weekends, when the professors are not available. Therefore, in January 2013, five instructors at South University started using online discussion forums to answer students’ questions and/or concerns. Students are using these discussion forums to answer each other’s questions. The online discussion forums allow professors to answer students’ questions at their own convenience. In this presentation, I would like to share the results of this intervention. In addition, I will share some of the strategies to keep students engaged outside the classroom.

Ezra “Bud” Brown, Virginia Tech
From Points of Inflection to Bones of Contention: The Birth of Finite Geometries, Block Designs, and Normed Algebras
2:15 PM, Henson Hall, Room 103
In this talk, we give a brief tour of the birth and early development of finite geometries, combinatorial designs, and normed algebras. Arthur Cayley (1845), Jakob Steiner (1853) and Gino Fano (1892) are credited with the creation of (respectively) the 8-dimensional real normed algebra, certain block designs with block size 3, and the first finite geometry. During our tour, we learn about the truly ground-breaking work of Julius Plücker, John Graves, Wesley Woolhouse and Thomas Kirkman, work that anticipated Cayley, Steiner and Fano by one, 18, and 57 years, respectively.

Even better – from the speaker's point of view -- the tour begins with elliptic curves and ends with the (7,3,1) block design.

Randy Cone, Virginia Military Institute
Jon Scott, Montgomery College
Building a Community: The American Mathematics Competitions and A Panel Discussion
8:10 AM and 8:35 AM, Henson Hall, Room 103

Each year, the MAA encourages young mathematical talent from across the nation to participate in its middle school and high school mathematics contests. These competitions are known as the American Mathematics Competitions (AMC), where the AMC8 is designed for 8th-grade competitors and the AMC10/12 tests are written for 10th and 12th-grade students respectively. Typically, students participating in the AMC tests do so at their own schools, though there is an established MAA program for colleges and universities to host the competitions. This short session will provide information about how some institutions in our MD-DC-VA Section have successfully become host institutions for the AMC tests, give suggestions to interested schools about how to become a host institution, as well as discuss a new MD-DC-VA Section initiative concerning the creation of an AMC5 for elementary school students.

Ming Fang, Norfolk State University
Application of Geometric Series to Financial Problems
2:40 PM, Henson Hall, Room 107

In this talk, we will discuss the application of Geometric Series to Mortgage/loan Amortization, Stock Valuation, and Monetary Economics.

Jennifer Galovich, St. John's University and Virginia Tech
Mathematical Modeling for the (Mathematically) Faint of Heart
8:10 AM, Henson Hall, Room 115

Modern problems in biology are extraordinarily complex; mathematical modeling has proven very useful in reducing that complexity. However the traditional tools of ODEs and PDEs, while useful for mathematicians, are not very accessible to biologists. Tools for analyzing discrete models, however, can be made very accessible, and at the same time rely on some deep and interesting results from modern algebra. I will describe some of these tools and how they can be used, and point to some of the deep mathematical results on which they are based.

Alexander Halperin, Lehigh University
H-linked Graphs with Prescribed Length
2:15 PM, Henson Hall, Room 107

Given a graph H and an integer sequence $D = \{d_{ij} \mid v_i v_j \in E(H), d_{ij} \geq 2\}$, a graph G is (H,D,e)-*linked* if every injective map f_1: $V(H) \rightarrow V(G)$ can be extended into an H-subdivision (f_1,f_2) in G such that each path $f_2(v_i v_j)$ has length within e of d_{ij}. If $e = 0$, then we say G is (H,D)-*linked*. Raising the minimum value of each integer $d_{ij} \in D$ to 14, we establish a sharp minimum degree condition for a large graph G to be $(H,D,1)$-linked. Additionally, we establish sharp minimum degree conditions for large graphs G to be $(H,D,1)$-linked and (H,D)-linked. Both results are proved using the Regularity Lemma (Szemerédi, 1978), the Blow-Up Lemma (Komlós, Sárközy, Szemerédi, 1994), and Williamson's theorem on panconnectivity (1977).

Spencer Hamblen, McDaniel College
Primes in Iterations of $z^d + c$
2:15 PM, Henson Hall, Room 113

Given a polynomial $f(z) = z$^$d + c$, what "percentage" of primes divide at least one of the infinite sequence $f(0)$, $f(f(0))$, $f(f(f(0)))$,...? We will examine some choices of d and c that ensure that almost all primes do not divide any term of the sequence.

Greg Hartman, Virginia Military Institute
APEX Calculus: The Open Source Calculus Textbook Project
9:25 AM, Henson Hall, Room 115

This presentation serves as an update of the ongoing work of collaboratively writing an open source Calculus textbook. The current edition is in use at VMI for Calc I & II, with Calc III scheduled for use this summer. Samples of the text will be available. Ample time will be given for discussion and suggestions to further the work.

Brian Heinold, Mount St. Mary's University
Some Different Applications of Logarithms
8:10 AM, Henson Hall, Room 107
Over the last year or so I have been collecting a variety of applications of logarithms for use in teaching. Many, if not most, should be different from the ones you may be familiar with. The applications are both interesting and useful for any class that introduces logarithms.

Ilhan Izmirli, George Mason University
Wittengstein was a Social Constructivist
3:05 PM, Henson Hall, Room 107
In this paper our main objective is to interpret the major concepts in Wittgenstein's philosophy of mathematics from a social constructivist point of view in an attempt to show that this philosophy is still very relevant in the way mathematics is being taught and practiced today. We start out with a brief discussion of radical constructivism followed by a rudimentary analysis of the basic tenets of social constructivism. We observe that, the social constructivist epistemology of mathematics reinstates mathematics, and rightfully so, as "… a branch of knowledge which is indissolubly connected with other knowledge, through the web of language" (Ernest 1999), and portrays mathematical knowledge as a process that should be considered in conjunction with its historical origins and within a social context. Consequently, like any other form of knowledge based on human opinion or judgment, mathematical knowledge has the possibility of losing its truth or necessity, as well. In the final section, we emphasize the connections between social constructivism and Wittgenstein's philosophy of mathematics. Indeed, we argue that the apparent certainty and objectivity of mathematical knowledge, to paraphrase Ernest (Ernest 1998), rest on natural language.

Emek Kose, St. Mary's College of Maryland
Geometry of Discrete Curves, Part III: Geometric Splining
3:05 PM, Henson Hall, Room 115
Please see the abstract for Part I of this talk under "Dan Carroll" in Student Abstracts for more details.

Brian Lins, Hampden-Sydney College
e in a Box of Cereal
3:05 PM, Henson Hall, Room 111
How would you explain the mathematical constant *e* to someone without a background in calculus? I will tell the story of how I found *e* in a box of cereal, and how my daughter found it in a pile of socks. I will also describe where *e* can be found on a logarithmic scale and in the pattern of the prime numbers.

Keith Mellinger, University of Mary Washington
Eigentriads – A Musical Offering
3:05 PM, Henson Hall, Room 103
Using a natural association of musical triads (3-note chords) with vectors over the ring Z_{12}, we identify the triads used in Western music that correspond to eigenvectors. Such triads we call eigentriads, and we show that these special triads are none other than augmented and diminished triads. We offer both an explanation of why this is so, and an interpretation of the meaning of an eigentriad in the context of a musical composition. This work is joint with Bud Brown and Alissa Crans.

Aurelia Minut, United States Naval Academy
Nonlinearity in Fiber Bragg Gratings
8:35 AM, Henson Hall, Room 111

In this talk, we show that the solution of linear coupled mode equations (LCME) is a good approximation to that of the nonlinear coupled mode equations (NLCME) for small times, provided that the nonlinearity is weak. We illustrate our findings in numerical examples. This work is motivated by the need to examine the behavior of a fiber Bragg grating when large amplitude light is propagated in the fiber.

Edwin O'Shea, James Madison University
The Utility and Practice of the Daily Quiz
9:25 AM, Henson Hall, Room 107
Inspired by his colleagues' use of a daily quiz in their calculus classes, the speaker will reflect on his own first experiences with such a regime. Practical tips for effective implementation and timely grading of said quizzes will be provided.

Cherng-Tiao Perng, Norfolk State University
Meta-Walk: A Journey of Discovery and Rediscovery
9:00 AM, Henson Hall, Room 113
(Joint work with Fred Lunnon) Our work is based on factorization of Lipschitz quaternions. Given an odd prime p (for simplicity, the case of the prime 2 can be studied separately), there are $p+1$ equivalence classes of prime quaternions of norm p. It can be shown that there is a one-to-one correspondence between the set of equivalence classes of prime quaternions of norm p and the set of projective solutions of the conic $x^2 + y^2 + z^2 = 0$ over the finite field $\boldsymbol{F}_p$. Since there are $p+1$ points on this conic and this conic is isomorphic to the projective line over $\boldsymbol{F}_p$, there should be a geometric connection between the set of equivalence classes of prime quaternions of norm p and the set of points on the conic. Conway and Smith called the factorization $PQ=Q'P'$ a *meta-commutation*, where P,P' (resp. Q,Q') are of norm p (resp. of norm q), and p,q are distinct prime numbers. We slightly generalize the situation: fixing a Lipschitz quaternion Q of norm not divisible by p, and for a prime quaternion P of norm p, we have the factorization $PQ=Q'P'$. We define a *meta-walk* from P to P' via the above relation; this is well-defined up to left association by the 8 units of Lipschitz quaternions. Previously we have investigated the meta-walk: the result of the meta-walk can be a fixed class $\{P\}$, or form a cycle consisting of more than one class, where the cycle length is divisible by $p+1$, $p-1$ or p. This can be shown to depend on $||\mathrm{Im}\ Q||$, namely the cycle length is divisible by $p-a$, where $a := (-||\mathrm{Im}\ Q||/p)$ is the Jacobi symbol. Recently we elucidated these patterns by working out the meta-walk as a projective correspondence (an automorphism) on the conic. Specifically, we found that the meta-walk defined by Q is up to scaling given by the Euler-Rodrigues formula (and reduction modulo p), when we associate each class of P with a point on the conic $x^2 + y^2 + z^2 = 0$ over $\boldsymbol{F}_p$. It follows that every meta-walk gives an orthogonal transformation on the conic. Furthermore, we exhibit three explicit types of walks (elliptic, hyperbolic, and parabolic) and show that they generate the full automorphism group $SO_3(\boldsymbol{F}_p) \approx PGL_2(\boldsymbol{F}_p)$ of the conic. An immediate corollary is that all cycle lengths greater than one of the meta-walk are equal.

Adrian Rice, Randolph-Macon College
Commutativity and Collinearity: A Fundamental Connection Between Pappus and Diophantus
2:40 PM, Henson Hall, Room 103
Pappus and Diophantus had almost nothing in common, apart from the fact that they were mathematicians working in or around Alexandria in roughly the 3rd-4th century AD. Moreover, the geometry of Pappus and the number theory of Diophantus had no mathematical connection whatsoever ... or so one might think. The fact is that a link always existed, but mathematics had not developed sufficiently for anyone to see it. Indeed it went unnoticed for well over 1500 years until the end of the 19th century, with the publication of two groundbreaking but again ostensibly unrelated works by two German mathematicians, Adolf Hurwitz and David Hilbert. In the interim, mathematics had changed out of all recognition, with the creation of numerous new mathematical subjects and disciplines, without which the connection might never have been noticed in the first place. This talk examines the chain of mathematical events that led to the discovery of this remarkable connection whereby two seemingly disjoint areas of mathematics were eventually found to be intimately related.

Robert Sachs, George Mason University
Beyond BC Calculus: An Inquiry-Driven Calculus Enrichment Course

2:40 PM, Henson Hall, Room 113

Last summer I taught this course for the first time for high school students who had completed BC calculus. I will do it again this summer. We chose topics based on their input and then explored them in an interactive inquiry style. I will describe the topics and student reaction.

Bonita Saunders, National Institute of Standards and Technology
Bruce Miller, National Institute of Standards and Technology
Marjorie McClain, National Institute of Standards and Technology
Daniel Lozier, National Institute of Standards and Technology

DLMF Live! Tables: NIST Digital Library of Mathematical Functions Tables Project

9:25 AM, Henson Hall, Room 103

In the past, researchers and mathematical software designers often used published tables of elementary and special function values for numerical exploration, software validation, algorithm development, or graphics production. The tables were sometimes tedious to use and suffered from severe drawbacks such as low precision, restriction to real arguments, a limited number of arguments and parameters, and an inability to accommodate alternate function definitions. This talk discusses NIST's collaboration with the University of Antwerp to address these problems by developing an online system where users can create high precision tables of elementary and special function values with an accuracy certification.

Amy Shell-Gellasch, Hood College

When a Number System Loses Uniqueness: The Case of the Maya

2:40 PM, Henson Hall, Room 111

The Maya of Central America had a very complex number system. Their modified base-20 system evolved out of their astronomical observations of Venus and their calendrics. The interesting consequence of this modified system is that is loses uniqueness. This talk will give a short overview of the Mayan number system and explain in what cases it does not express numbers uniquely. This material is interesting from a mathematical point of view and can also be used in a number of math and math history courses.

Thomas Sibley, St. John's University and College of St. Benedict

When is a Cube like a Tetrahedron?

9:25 AM, Henson Hall, Room 111

A colorful use of graphs allows us to infuse the formal beauty of abstract algebra into the visual beauty and intuition of geometry. Geometric analogs to algebraic concepts give different insights of familiar geometric objects and suggest new areas to explore. For example, homomorphisms, substructures and direct products link a cube to a tetrahedron.

Wendy Hageman Smith, Longwood University

Leveraging NOVA and BBC-produced Popular Math Videos into the History of Math Curriculum to Enhance Accessibility and Students Engagement

8:35 AM, Henson Hall, Room 107

It's a rare student who's jazzed about both mathematics and history, so teaching the history of math can be a challenge. Most textbooks are too advanced for our undergraduates, and assume a level of interest that is usually lacking. I have developed a set of techniques to meet the challenge of engaging students in the mathematics through the drama of its historical development, with special focus on the evolution of our understanding of mathematical infinities. In particular, we can leverage the videos made by popularizers of mathematics, integrating them topic by topic into the textbook/syllabus and classroom exploration. I will present examples of this integration.

Ivan Sterling, St. Mary's College of Maryland

Geometry of Discrete Curves, Part II

2:40 PM, Henson Hall, Room 115

Please see the abstract for Part I of this talk under "Dan Carroll" in Student Abstracts for more details.

Madelyn Windley, George Washington University
African-Americans in Mathematics: Geniuses Lost in the Shuffle
8:10 AM, Henson Hall, Room 113
Black History IS American History. This is a phrase I often hear some Blacks/African-Americans say during the month of February (i.e. Black History Month). The point is to say that the contributions that blacks have made throughout the history of America is just as important as the contributions that whites and other races have made. If we agree that their contributions are important, then we should talk about it! This presentation will be about African-Americans such as Benjamin Banneker and Thomas Fuller who sort of beat the odds when it came to their knowledge of mathematics. With an introduction to the history of mathematics in Africa as researched by the University at Buffalo, we will discuss how even though it was believed that Africans and their descendants were incapable of learning, some excelled in mathematics without a formal education, during a time when it was illegal to teach a slave how to read and write.

Godfred Yamoah, Norfolk State University
A Time Control Scheme for Richards' Equation
8:35 AM, Henson Hall, Room 113
Adaptive temporal integration is needed in the solution of Richards' Equation (RE) for several reasons. The solution often results in the formation of sharp fronts. Thus a small time step is required duration the formation of the front. A fixed time step method is not computationally efficient. Also the user has no control over the accuracy of the solution. Variable time-stepping based on error control improves both the accuracy and robustness of the code and allows the user to supply an error tolerance for desired accuracy. We consider a basic first-order one-step method which is derived under the assumption that the nonlinearity of RE is smooth. This allows the next time step size to be expressed as a function of the local truncation error in the current step.

Student Abstracts by Author

Traymon Beavers (Sophomore), James Madison University
Graphs from Beyond the Grave: A Self-Stabilizing Algorithm for Double Domination
2:40 PM, Henson Hall, Room 123
Developed by Edsger Dijkstra, self-stabilizing algorithms play a key role in fault tolerant distributed computing. They account for the inevitable fact that all computers can fail and still allow a network to function. We propose two serial self-stabilizing algorithms that result in a minimal double dominating set of nodes. In 2000, Harary and Haynes defined a doubly dominating set in a graph $G = (V;E)$ as one comprised of all verticies that dominate a node at least twice. From modeling networks, assisting in decision making, to helping to save the world in the event of a zombie apocalypse, this variation on standard domination has many uses. Both algorithms are built for any finite but arbitrary graph; one specialized for C_n and one for $G_{m,n}$.

Lisa Borum (Senior), Randolph-Macon College
Pebbling Samurai Sudoku and Chessboard Graphs
9:00 AM, Henson Hall, Room 109
A pebbling move is defined as removing two pebbles from a vertex and placing one pebble on an adjacent vertex. The pebbling number of a graph is defined to be the smallest number of pebbles such that when placed in any configuration there is a sequence of pebbling moves available to get to any empty vertex. Two classes of graphs are defined, Samurai Sudoku and Chessboard, and lower bounds are presented for pebbling numbers of variations of these graphs.

Sara Brooks (Junior), Christopher Newport University
Ashley Marzzarella (Graduate Student), Christopher Newport University
Alyssa Walzak (Freshman), Christopher Newport University
Gretchen Jewell (Junior), Christopher Newport University
Monster Math Club
3:05 PM, Henson Hall, Room 109

The Monster Math Club was designed to inspire and encourage middle school students, especially young women and minorities, to pursue mathematics and to learn about careers in which mathematics play a role. The program links math majors from Christopher Newport University (CNU) and professionals from mathematics-related fields with a group of 6th – 8th grade minority students who attend an after school care program at the Hampton YMCA. The main objective is to encourage middle school students who have an interest in mathematics to take the more challenging math classes as they move towards high school. Through the weekly interactions during the school year, the students have participated in enrichment activities, visited museums and math-related exhibits and received guest speakers. This was all made possible through a Tensor-SUMMA award from the Mathematical Association of America.

Dan Carroll (Junior), St. Mary's College of Maryland
Geometry of Discrete Curves, Part I
2:15 PM, Henson Hall, Room 115
We will give three presentations on the geometry of discrete curves. In particular we will survey the literature and attempt to clarify the many different and conflicting definitions of discrete curvature and discrete torsion. We will in particular emphasize three notions of curvature based on the inscribed, circumscribed and centered circles of a regular *N*-gon. In Part I, with Dan Carroll presenting, we will give our pleasing version of discrete Frenet equations. In Part II, with Ivan Sterling presenting, we will discuss discretizing a smooth curve using the inscribed, circumscribed and centered definitions of curvature. Finally, in Part III, with Emek Kose presenting, we will discuss splining a discrete curve in a inscribed, circumscribed and centered way. This is joint work with Eleanor Hankins.

Heather Cook (Junior), Roanoke College
Surviving an Outbreak of Zombie Dice: An Optimal Play Strategy
2:15 PM, Henson Hall, Room 109
We will discuss the game of "Zombie Dice," published by Steve Jackson Games. Game play will be explained, and then we will investigate a model for deciding whether or not to continue rolling if given the opportunity. Examples will be shown to highlight the game and its strategy.

Greg Gibbons (Senior), Virginia Military Institute
Alex Lin (Sophomore), Virginia Military Institute
Narathip Khanhansuk (Sophomore), Virginia Military Institute
Situation Theory and Mathematical Modeling for Strategic Freshwater Management
9:25 AM, Henson Hall, Room 109
Freshwater ecosystems provide essential services for human populations and are home to the greatest concentration of biodiversity on Earth. As abuse and degradation of these ecosystems increase from factors such as pollution and climate change, countries around the world have strived for a successful water strategy focusing on methods of storage and movement, de-salinization, and conservation. Using Russia as our subject, our research observed poor water quality that stems directly from the regulation of poor fresh water resource management. In this paper, we developed a feasible, uniform, and descriptive mathematical model that has potential in serving as a framework for capturing nearly all the complexities of fresh water resource management. Our model is based on an information-ow view, referred to as situation theory, that involves layers upon layers of different situations for the country's fresh water. It allows researchers, government agencies, and other interested parties to analyze data of different 'objects' (i.e. the workers, de-salinization technology, location of water sources) in a given problem and formally determine an optimal solution.

Eleanor Hankins (Senior), St. Mary's College of Maryland
Constructing Hedgeworms: A Union of Line Segments through each Point of Concave Curve with Hausdorff Dimension One
9:00 AM, Henson Hall, Room 115
A Besicovitch set is a compact set in R_n that contains segments of unit length in every direction. Besicovitch discovered there exist Besicovitch sets in the plane with Lebesgue measure zero, implying Besicovitch sets of zero measure exist in higher dimensions as well. The famous Kakeya conjecture states that the Hausdorff dimension of a Besicovitch set in R_n

must be *n*. In 1971, Davies proved this for the 2-dimensional case. In our research, we explore smaller, related problems. We can project a Besicovitch set in R_2 to a set consisting of a line with line segments through each point. Since projection cannot increase Hausdorff dimension, this set must have Hausdorff dimension 2 by Davies' result. Our research focuses on the construction of a curve in 2 dimensions with lines through each point, which has dimension less than 2. In this presentation, we prove the existence and validity of such a construction and prove it has Hausdorff dimension 1.

Joshua Kaminsky (Senior), St. Mary's College of Maryland
A Different Type of Polygon
8:10 AM, Henson Hall, Room 111
There are a number of generalizations of convexity, and some of these lead to a generalized notion of convex polygons. We focus on the concepts of α-convexity and reach α to construct the class of α -polygons. The presentation will motivate these new polygons and present some generalizations of traditional properties.

Michael Ladas (Sophomore), Montgomery College
Modeling the Belousov-Zhabotinsky Reaction
8:10 AM, Henson Hall, Room 109
The Belousov-Zhabotinsky Reaction is an oscillating chemical reaction. I will compare a model programmed in MATLAB to the reaction itself. The primary goal is to model the concentrations, of each chemical species, as the BZ reaction progresses towards equilibrium. Additionally, I look to identify the rate constants of the reaction for four different initial conditions.

Meg Protzman (Sophomore), McDaniel College
Functional Dependence Between Boolean Variables
2:15 PM, Henson Hall, Room 123
Functional dependence relation between sets of variables has been axiomatized by Armstrong in 1974. We have investigated this relation between single variables in a special case when all variables have the same fixed number of values. In the Boolean case of just two values we were able to give a complete axiomatization. In a more general case of 3 values we found non-trivial properties, but were not able to prove completeness.

Kristin Strand (Senior), Virginia State University
Existence of Geometric Triple Systems
2:40 PM, Henson Hall, Room 109
Let G be an abelian group, $k \in$ G and let ϕ: G → Π be an injective mapping from G into the Euclidean plane Π such that no four points in ϕ(G) are collinear. If for each three element subset {*a*, *b*, *c*} of G such that $a + b + c = k$, the corresponding points {ϕ(*a*), ϕ(*b*), ϕ(*c*)} are collinear, then we call the set of points ϕ(G) and the corresponding lines, a *geometric triple system on G with sum k.* In this presentation, we will show that geometric triple systems exist if the base group G is Z_n or Z, and that they can be constructed on cubic curves.

Anna Tuck (Senior), UNC-Greensboro
Population Dynamics Model with Logistic Growth, Weak Allee Effect, and Grazing on an Interior Patch
9:00 AM, Henson Hall, Room 111
We study the positive solutions to the steady state reaction diffusion equations with Dirichlet boundary conditions of the form:

$$-u'' = \begin{cases} \lambda\left[u - \frac{1}{K}u^2 - c\frac{u^2}{1+u^2}\right]; & x \in (L, 1-L), \\ \lambda\left[u - \frac{1}{K}u^2\right]; & x \in (0, L) \cup (1-L, 1) \end{cases}$$
$$u(0) = 0 = u(1)$$

and

$$-u'' = \begin{cases} \lambda\left[u(u+1)(b-u) - c\dfrac{u^2}{1+u^2}\right]; & x \in (L, 1-L), \\ \lambda[u(u+1)(b-u)]; & x \in (0, L) \cup (1-L, 1) \end{cases}$$
$$u(0) = 0 = u(1)$$

Here, λ, b, c, K, and L are positive constants with $0 < L < 1/2$. These type of steady state equations occur in population dynamics; the first model describes logistic growth with grazing, and the second model describes weak Allee effect with grazing. In both cases, u is the population density, $1/\lambda$ is the diffusion coefficient, and c is the maximum grazing rate. These models correspond to the case of symmetric grazing on an interior region. Our goal is to study the existence of positive solutions. Previous studies when the grazing was throughout the domain resulted in *S*-shaped bifurcation curves for certain parameter ranges. Here, we show that such *S*-shaped bifurcations occur even if the grazing is confined to the interior. We discuss the results via a modified quadrature method and *Mathematica* computations.

Caroline VanBlargan (Junior), St. Mary's College of Maryland
Finding a Mirror and Projection in Mathematical Imaging
8:35 AM, Henson Hall, Room 115
A catadioptric sensor is an optical device that uses a camera and a mirror as a reflective surface to map light rays from the world to an image. The standard problem in catadioptric sensor design is to find a mirror surface that induces a specified mapping, where the projection from the image plane is fixed. It has been shown, however, that such a mirror will not always exist. Here we will present a new technique that allows one to solve for both a projection and mirror surface together so that the mapping is realized. This results in a non-linear system of partial differential equations which must be solved numerically.

Michael Varghese (Junior), Montgomery College
Deflection of a Beam
3:05 PM, Henson Hall, Room 123
ODE's are used in various engineering applications. We will look at a second-order linear ODE used to calculate beam deflections and solve it using finite differences. The beam will be simply supported and have a constant, uniformly distributed load.

Carrie Winterer (Junior), St. Mary's College of Maryland
What do Math and LEGOs Have in Common?
8:35 AM, Henson Hall, Room 109
Legend has it that Queen Dido used strips of an ox's hide to solve the classical isoperimetric problem, which involves finding the maximum area that can be enclosed given a fixed perimeter. Using LEGO bricks we describe our solution(s) to a simpler analogous question: What is the smallest number of non-overlapping 1 x 1 bricks needed to enclose a given area? Our problem assumes that all areas are integral, and a 1 x 1 square corresponds to one unit of area. Standard optimization strategies combined with discrete mathematics and number theory allow us to show that the smallest perimeter needed to enclose A units of area is $P(A) = 2\left\lceil 2\sqrt{A}\right\rceil + 4$. This question lends itself to interesting generalizations. Polyominoes can be used to similarly discretize the isoperimetric problem, however, when using LEGO blocks both area and perimeter have two dimensions: length and width. This aspect is lost when using polyominoes. Other generalizations include using different brick-shapes, imposing various weighting constraints, and the formulation of a "LEGO double bubble" problem comparable to the one solved by Dr. Frank Morgan. No background will be assumed. [Joint with: Isabel Guadarrama, Emmanuel Daring, Samantha Sprague]

Student (Poster) Abstracts by Author

Alexander Bracco (Senior), Virginia Military Institute
Brian Graham (Senior), Virginia Military Institute
Nam Nguyen (Senior), Virginia Military Institute
The Ultimate Brownie Pan

2:10 PM – 3:30 PM, Henson Hall Food Court
Our poster will demonstrate how to choose the most efficient brownie making pan that also provides the best quality brownies.

Kathy Dunning (Senior), Salisbury University
Samim Manizade (Sophomore), Salisbury University
Ross Tavjar (Sophomore), Salisbury University
I Think, Therefore I Bake: Building a Better Brownie
2:10 PM – 3:30 PM, Henson Hall Food Court
For a given oven size what pan shape should be used so that brownies are baked evenly and the number of brownies baked per oven cycle is as large as can be? This is the question posed in COMAP's 2013 Mathematical Contest in Modeling. Circular pans have the most equal heat distribution, but make very inefficient use of oven space. Rectangular pans make the most efficient use of oven space, but have an uneven heat distribution (the corners tend to reach higher temperatures for longer periods of time). Since it is not possible to satisfy both conditions simultaneously, the next best thing is to design indices that measure the trade-off between heat distribution and space utilization. We design such indices and then optimize the index over pan shapes. To arrive at a solution we simplify the model to consider only two dimensional brownie pans. We then simulate heat flow in the brownie pan using the method of finite differences via MATLAB, and display the results using Mathematica's graphical interface. We find that a stadium shaped pan (a hybrid of a circle and a rectangle) is optimal with respect to our indices.

Ryan Hillman (Senior), Virginia Military Institute
Iain McPherson (Senior), Virginia Military Institute
James Snyder (Senior), Virginia Military Institute
Saudi Arabia Water Strategy
2:10 PM – 3:30 PM, Henson Hall Food Court
Water is the foundation for life on this planet, which is why the availability of potable water throughout the world is of the utmost importance. Saudi Arabia, like many other nations, is currently experiencing a water crisis wherein current consumption rates could leave the country with severe water shortages, in turn bringing about significant economic and domestic implications. In this paper we describe the development of a mathematical model that addresses the Saudi Kingdom's water strategy with respect to three primary categories: storage/movement, desalination, and conservation. Using this model, we develop an effective, feasible, and cost-efficient water strategy for 2013 to meet the projected needs in 2025. We develop a conceptual derivative model with the purpose of increasing wastewater treatment, desalination, and surface water withdrawal, which leads to a decrease in nonrenewable groundwater usage. We then make specific water strategy recommendations that stem from the model. These recommendations include irrigation system changes, pipeline reparations, technological advances in desalination, and more. Such recommendations, if applied immediately, will lead to a more sustainable water supply in the near future by increasing yearly water yields.

Samim Manizade (Sophomore), Salisbury University
Computational Modeling of Robotic Arm Kinematics
2:10 PM – 3:30 PM, Henson Hall Food Court
This project focused on the development of mathematical techniques to model the motion of a robotic arm consisting of multiple rigid linkages connected by joints which move in a variety of ways. The end of the arm contains a hand which, in practice, could be used to manipulate real world objects. A variety of arms are modeled using the computer algebra system Mathematica, from a very simple one-joint-one-segment arm to an arm with four joints and segments as well as an adjustable end effector. Mathematical techniques are then used to examine the relationship between the joint space of allowable joint settings and the configuration space which describes the possible hand positions. For certain arms, the Forward Kinematic problem is solved in order to determine what points are reachable by certain adjustments of the arm. For others, the Inverse Kinematic problem yields solutions that identify which configurations of a given arm will allow it to reach a certain point. Mathematica proves a powerful tool for solving the complicated set of equations involved in the Inverse Kinematics problem, and so a Wolfram Demonstrations Project is being created to interactively

solve the inverse kinematic problem. Real-world demonstrations of the concepts dealt with in Mathematica are achieved through the use of Lynxmotion's AL5a arm.

Michael Spickard (Senior), Virginia Military Institute
Eric Summers (Senior), Virginia Military Institute
Caitlin Bradly (Junior), Virginia Military Institute
The Ultimate Brownie Pan
2:10 PM – 3:30 PM, Henson Hall Food Court
The Virginia Military Institute (VMI) requires that all cadets complete a capstone course as a graduation requirement. Specifically, the Applied Mathematics Department requires all cadets within the major compete in the annual Mathematical Contest in Modeling (MCM) as a requirement of the capstone course. Our group of three cadets was one of several groups that represented VMI in the 2013 MCM this past February. This paper describes our solution to one of the two problems presented to us. Our chosen problem concerns being able to bake pans of brownies that are not overcooked around the edges. Our goal is to find a pan that best optimizes oven space and evenly distributes heat while baking. We develop a model to show the distribution of heat across the outer edge of a brownie pan. Our objectives are to first maximize the number of pans that can fit in a standard kitchen oven and to maximize the even distribution of heat for the pan. Using GeoGebra, downloadable interactive geometry software, we are able to model the heat distribution across the outer edges of the pan. When we optimize the number of pans that fit in the oven, along with our necessary condition of even distribution of heat, and find that the square pan best fits our needs.

Andrew Yang (Senior), McDaniel College
The Circle Pairing on Elliptic Curves
2:10 PM – 3:30 PM, Henson Hall Food Court
The circle pairing is defined to be a function from pairs of points on an elliptic curve defined over a real quadratic field ($[P,Q]$:$\boldsymbol{Q}(\sqrt{d})$ where P and Q are points on the elliptic curve and d is a square free positive integer) to $\boldsymbol{Q}/\boldsymbol{Z}$. Not much is known about the circle pairing, but it is conjectured to capture essential information about the elliptic curve, leading to an analogue of the Birch and Swinnerton-Dyer conjecture.