Occupy Phase Space:
The Mathematics of Dissent and Suppression

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In recent years, mathematical models from biology have been adapted to model the various interactions between regimes and opposition groups, or between security forces and terrorist organizations. These models predict some surprising results, with policy implications. We'll examine some of these models; see what they tell us about the interaction between groups; and consider extensions.

The Problem

- Two groups: Regime and Opposition

- Opposition uses violent or peaceful means

- Regime suppression works...

- ...unless it fails.

	Suppression Works	Suppression Fails
Violent	Iraq (1991)	Libya (2011)
Peaceful	Burma (2007)	India (1947)

"Political power grows out of the barrel of a gun" (Mao Zedong)

"Pure goals can never justify violent action" (Gandhi)

"Let them hate, so long as they fear" (Caligula)

"Let them eat cake" (Marie Antoinette)

Lichbach (1987)

- Regime-Opposition as Constrained Optimization

  - Gain from regime as objective function.

  - Opposition pays cost for conflict activities.

  - Find balance of strategies that minimizes cost.

- Regime Coercion

  - Changes cost function.

  - Conclusion: Suppression of more effective activity leads to more conflict events.

  - Consequence: In a liberal democracy, peaceful methods are more effective, so suppression of protests and demonstrations is counter-productive.

Tsebelis and Sprague (1989)

- Coercion and Revolution

- Predator-Prey Interpretation

  - Prey = Dissent activities (R)

  - Predator = Coercion by regime (C)

$$\frac{dR}{dt} = -fR + gC + h \qquad \frac{dC}{dt} = mR - nC + k$$

- Do they work?

  - Measuring "dissent activities"

  - Measuring "coercion"

Francisco (1996)

- Predator-Prey Models...With Data

  - Coercion measured by deaths/injuries/arrests

  - Dissent measured by protests, or terrorist actions.

  - Statistically significant correlations

- Signs significant

  - Germany: Regime coercion leads to *more* dissent.

  - Northern Ireland: Regime coercion little effect on terrorist activity (stable fixed point).
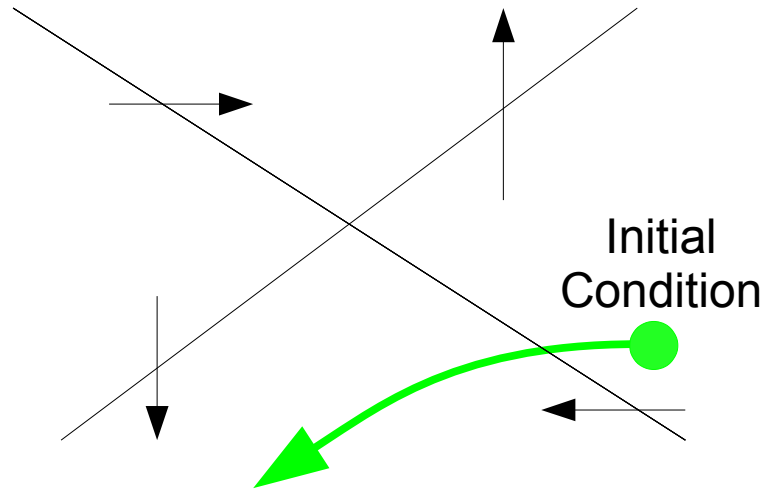
Guitfraind (2009)

- ## Leaders (L) and Followers (F)

  - ### Promotion from follower to leader

  - ### Recruiting based on prestige function gL + F.
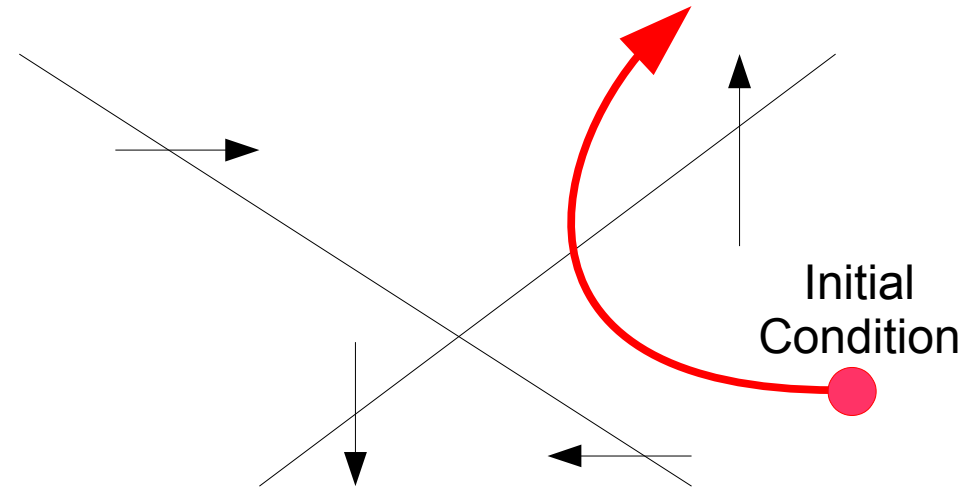
  - ### Leslie-type model

$$\frac{dL}{dt} = -aL + bF - h \qquad \frac{dF}{dt} = m(gL + F) - nF - k$$

- ## Counter-terrorist actions affect h, k

- ## *Type* of fixed point unchanged.

- ## *Location* altered (isoclines shifted).

Leave Well Enough Alone?

Initial Condition

Original (No Intervention, group decays)

Initial Condition

Perturbed (Intervention, group grows)

- Perturbation shifts isoclines.

- Could turn "decay" into "growth" (or vice versa).

- Translates into policy guidelines?

Further Considerations

- Opposition in two factions: Doves and Hawks.

- Simple system (no sign assumptions)

$$\frac{dx}{dt} = ax + by + h \qquad \frac{dy}{dt} = cx + dy + k$$

  - Spontaneous growth rates: h, k

  - Recruiting and defection rates: a, b, c, d (*any* sign).

  - Assume stable fixed point.

- Assumption of ignorance

  - Equations accurate...but parameters unknown.

  - What's the best course of action?

Regime Suppression

- Regime suppression

  - vs. doves or hawks: reduces constant

  - Changes *location* of fixed point, but not *type*.

  - Which leads to better result?

- Key question: What happens to fixed point?

  - Decay: doves, hawks decrease

  - Moderation: doves increase, hawks decrease

  - Growth: doves, hawks increase (but proportionally).

  - Radicalization: doves decrease, hawks increase

The Martyrdom Effect

- Assume stability
  - Jacobian is $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, so stability has a + d < 0, ad - bc > 0.
- Location of fixed point determined by a, b, c, d, h, k
  - Cramer's rule gives $\bar{x} = \dfrac{kb - hd}{ad - bc}$, $\bar{y} = \dfrac{hc - ak}{ad - bc}$
  - Partial derivatives gives:

$$\frac{\partial x}{\partial h} = \frac{-d}{ad - bc} \qquad\qquad \frac{\partial y}{\partial h} = \frac{c}{ad - bc}$$

$$\frac{\partial x}{\partial k} = \frac{b}{ad - bc} \qquad\qquad \frac{\partial y}{\partial k} = \frac{-a}{ad - bc}$$

- Regime can suppress doves (reduce h) or hawks (reduce k). Which yields better outcome?

Gandhi or Mao?

- Eight sign combinations consistent with stability:

a, d	b, c	vs. Doves	vs. Hawks
$a, d < 0$	$b, c < 0$		
	$b < 0 < c$		
	$c < 0 < b$		
	$b, c > 0$		
$a < 0 < d$	$b < 0 < c$		
	$c < 0 < b$		
$d < 0 < a$	$b < 0 < c$		
	$c < 0 < b$		

Gandhi or Mao?

- Case analysis (yellow = undesirable results)

a, d	b, c	vs. Doves	vs. Hawks
$a, d < 0$	$b, c < 0$	R	M
	$b < 0 < c$	D	M
	$c < 0 < b$	R	D
	$b, c > 0$	D	G
$a < 0 < d$	$b < 0 < c$	M	M
	$c < 0 < b$	G	D
$d < 0 < a$	$b < 0 < c$	D	G
	$c < 0 < b$	R	R

- Better to act vs. Hawks.

Gandhi or Mao?

- Avoid Radicalization

a, d	b, c	vs. Doves	vs. Hawks
$a, d < 0$	$b, c < 0$	R	M
	$b < 0 < c$	D	M
	$c < 0 < b$	R	D
	$b, c > 0$	D	G
$a < 0 < d$	$b < 0 < c$	M	M
	$c < 0 < b$	G	D
$d < 0 < a$	$b < 0 < c$	D	G
	$c < 0 < b$	R	R

- Better to act vs. Hawks.

Gandhi or Mao?

- Hope for moderation

a, d	b, c	vs. Doves	vs. Hawks
$a, d < 0$	$b, c < 0$	R	M
	$b < 0 < c$	D	M
	$c < 0 < b$	R	D
	$b, c > 0$	D	G
$a < 0 < d$	$b < 0 < c$	M	M
	$c < 0 < b$	G	D
$d < 0 < a$	$b < 0 < c$	D	G
	$c < 0 < b$	R	R

- Better to act vs. Hawks.

Gandhi or Mao?

- Worse outcomes: vs. Doves (R, G), vs. Hawks (M, G)

a, d	b, c	vs. Doves	vs. Hawks
$a, d < 0$	$b, c < 0$	R	M
	$b < 0 < c$	D	M
	$c < 0 < b$	R	D
	$b, c > 0$	D	G
$a < 0 < d$	$b < 0 < c$	M	M
	$c < 0 < b$	G	D
$d < 0 < a$	$b < 0 < c$	D	G
	$c < 0 < b$	R	R

- Better to act vs. Hawks.

Conclusions

- "Who lives by the sword..."

  - Better to act vs. Hawks

  - Leave peaceful faction alone!

- Next steps

  - Model regime apathy?

  - Better ODEs for modeling regime-opposition interaction?

  - Tests of models?

  - Regime coercion as change of *coefficients*?

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