# MD-DC-VA Section MAA Spring 2008 Meeting at EMU and JMU

# **Faculty Contributed Paper Abstracts**

Asterisks mark papers which the presenter has indicated as recommended for students.

# E.T. Brown, James Madison University

# A survey of mathematical art \*

We present a survey of mathematical art, with the category being broadly construed. It is hoped that these examples and references will be of use to faculty teaching mathematics at several levels, and mathematically motivated artists.

# Ezra Brown, Virginia Tech

# Elliptic Curves across the Curriculum \*

Elliptic curves are among the most beautiful and profound objects in all of mathematics. But what do they have to do with trigonometry, calculus, vector geometry, differential equations, abstract algebra, complex analysis, topology, number theory, cryptography, algebraic geometry, and undergraduate research? Come and find out!

# Kira Hamman, Penn State Mont Alto

# Mathematics and Democracy: A quantitative and political literacy course

Mathematics and Democracy is a general education quantitative literacy course that ties together a variety of QL topics by relating them to the democratic process: voting methods, apportionment, voting power, and fair division. Students in the course are encouraged to become more politically aware and to apply the mathematics they learn to a topic in this spectrum which particularly interests them. The course includes lectures by a historian, a political scientist, and a judge, and leaves students feeling that they have learned something which is actually relevant to their lives. I will talk about how the course was developed, the topics that are covered, and some successes and pitfalls.

# Chris Kennedy, Christopher Newport University

# Simple Lie and Associative Deep Matrix Algebras

We provide a "state of the art" talk about deep matrix algebras in both the associative and Lie settings, with the aim of exploiting the associative structure to explore the Lie structure. In the process, we will introduce several new simple Lie and associative algebras.

# Parviz Khalili, Christopher Newport University

# **On Matrix Representation of Linear Transformations \***

Given two different bases of an Euclidean space  $R^n$ , I will classify all linear transformations that have the same matrix representation in those two bases. Also if time permits I will give a short proof of the well known theorem: *Rowrank*(*A*)=*Columrank*(*A*).

# Yesem Kurt, Randolph College

# Non-commutative Cryptography \*

One of the main issues in cryptography is the encryption and decryption of private messages. A key exchange system allows the users to agree on a common key to encrypt and decrypt their messages. Common key exchange systems in use today work on commutative groups. This paper describes a new key agreement scheme that works over non-commutative structures, in particular over matrices.

# Aprillya Lanz, Virginia Military Institute

# Uniqueness implies existence for various classes of boundary value problems \*

In this presentation, we show uniqueness implies existence for  $y^{n}(n) = f(t, y, y', ..., y^{n}(n))$  subject to boundary conditions of the form  $L_{i}(y) = y_{i}$ , where  $L_{i} : C^{n}(n)$  ([a,b],R) to R are continuous linear functionals and *f* satisfies an appropriate growth condition.

# Stephen Lucas, James Madison University

# Integral approximations to pi with nonnegative integrands \*

We show how to find integrals with nonnegative integrands that evaluate to a fraction near pi. Some of these can be used to define new series for pi, where each term can add an arbitrary number of digits.

#### Betty Mayfield, Hood College

# Focus on National Meetings: Your opinion wanted

Are you the only person in your department who goes to the Joint Meetings? Have you always wondered what MathFest was all about? Do you have suggestions about how the national meetings could be improved? Whether you are a regular attendee or someone who rarely participates, the MAA's Strategic Planning Working Group on Meetings would love your input. This is a session in which YOU get to talk; please join us for a short conversation with your colleagues.

# Roland Minton, Roanoke College

## Hardy, Littlewood and Golf \*

Against the odds, both GH Hardy and JE Littlewood solved problems related to golf. The focus in this talk is on the odds of a steady golfer beating an erratic golfer of "equal" ability. Hardy's work on a similar problem adds a slice of history to this golfing adventure.

# Jack Pace, Southern Polytechnic State University

# Computer-Assisted Visualization of Complex Functions with Stereo Pairs \*

The graph of a function f on the complex numbers may be naturally regarded as a surface in real 4-space  $R^4$ . By projecting this surface onto a hyperplane of  $R^4$ , and then displaying a stereo pair of the projection, one may attempt to visualize the graph of the function f. Along with interactive rotations, this gives some insight into the properties of f. Examples of analytic functions, multivalued "functions", and other surfaces in *CXC* will be shown. Anaglyphic (red-blue) glasses will be provided for viewing the stereo pairs. Guests may bring their own popcorn.

# G. Edgar Parker, James Madison University

# Representing the Projectively Polynomial Functions Analytically within the Classical Algebra of Formal Power Series \*

P, the set of projectively polynomial functions has been shown to have wide applicability in computing solutions to both ordinary and partial differentiation and theory developed for P shows it to be, for +, , and o, a proper subalgebra of the set of real-analytic functions, A, which has been represented classically as a subalgebra of the set of formal power series. In this talk, we present, using the analytic properties of P, a representation of it within the classical algebra of formal power series. The representation eliminates some of the issues involved in dealing with the algebra of functions, most notably those concerning domain comparisons and intervals of convergence. The topological considerations afforded by the representation suggest possibilities for representing non-analytic functions with formal power series in a "natural" way.

# Robert Sachs, George Mason University

# The Lagrange Multiplier Rule as Seen by Linear Eyes \*

A local analysis of constrained optimization leads students to a better understanding of how the Lagrange multiplier rule arises. It is also consistent with how unconstrained optimization is usually described.

# James Sochacki, James Madison University

# Analytic Functions and Initial Value ODE's \*

It is well known that if the function *f* is analytic in a neighborhood of *A* then the solution *x* to the IVODE x' = f(x); x(0) = A is analytic in a neighborhood of 0. I will consider the properties of the limit function of the sequence defined by  $x_n' = f(x_n) \mid x_n(0) = A_n$ . Suppose  $f(x) = x^n f$  for some real number *r*. What are the differentiable properties to these solutions? If *f* is a polynomial what can you say about the solution *x*? In this talk I will discuss interesting differentiable properties of solutions to these IVODE's, the functions that are limits of solutions to these IVODE's and the computation implications when determining approximate numerical solutions.

# Ann Stewart, Hood College

# Project Math QUEST: Classroom Voting in Collegiate Mathematics Courses

Project Math QUEST is a three year project to both write and test classroom voting questions for use in differential equations and linear algebra courses. In classroom voting, the professor presents a multiplechoice question to the class, and then encourages the students to discuss the question in small groups with their peers. After voting individually on the correct answer, the professor then leads a class discussion of the results. Voting can be implemented in various ways, which include electronic hand-held "clickers". Study results and student survey responses indicate that this technique has a significant positive effect on participation, attitude and academic performance of students in the class. Project Math QUEST is administered by Holly Zullo, Kelly Cline, and Mark Parker of Carroll College, with funding from the National Science Foundation, and Hood College served as a question testing site during the 2007-2008 academic year.

# Ahlam Tannouri, Morgan State University *Mathematical Modeling: Monte-Carlo Simulation* \*

I will present different applications of Monte-Carlo simulation highlighting the law of large numbers.

## Bruce Torrence, Randolph-Macon College

## Beauty in Motion - Dynamic Visualization with Mathematica \*

Many mathematical ideas can be communicated visually. Interactive visualizations that respond in realtime to user input add fun to the mix, and often illuminate the ideas even further. In this talk, we will explore some easy-to-make dynamic visual environments that do exactly this. You'll leave ready to create dynamic visualizations of your own.

# Eve Torrence, Randolph-Macon College

# Self-Similarity in Origami Crease Patterns \*

The crease pattern of an orgami object is the pattern of lines that remains when the object is unfolded. There are four starting "bases" used in traditional origami. The crease patterns of these bases are repeated in many origami creations. With more complex folds come more repetitions of these patterns.

# Daniel Vasiliu, Christopher Newport University

# Constant Rank Linear Combinations of Matrices \*

Motivated by a non-convex minimization problem we study the case when any linear combination of a given set of matrices preserves the same rank. We explore necessary and sufficient conditions and some interesting examples.

# MD-DC-VA Section MAA Spring 2008 Meeting at EMU and JMU

# **Student Contributed Paper Abstracts**

# Jorge Bruno, Matthew DiGiosaffatte, and Benjamin Leard, James Madison University

# Ode to Sudoku (COMAP Presentation)

We present an algorithm that generates 2,903,040 Sudoku puzzles at a desired difficulty level. These puzzles are intended for human use, so trial-and-error solving techniques are unacceptable. Avoiding trial-and-error solving allows us to establish uniqueness of a solution. We also derive a difficulty metric *D* that can produce a difficulty value for a uniquely solvable puzzle in terms of any finite set of deterministic solving techniques. Metric analysis is given for the case where a puzzle can be solved using only the most simple deterministic technique, the Hidden One. We apply our algorithm to generate seven Sudoku puzzles of varying difficulty. These puzzles have unique solutions and are created using a short list of steps. This application illustrates the versatility and simplicity of our algorithm.

# Erin Casey, Randolph-Macon College

# What about the Cubic?

Every high school math student knows the general formula for solving a quadratic equation. Why is it then that no one knows how to solve a general cubic equation? This talk will use the history of algebra to try to answer this question. In particular, we will examine the historical developments that led to the discovery of the cubic formula in the 16th century.

# Andrew Chang, James Madison University

# Minimizing the Expected Value of the Variance of the X-Interprept in LS Regression

Chemists are usually interested in using the best spacing of concentrations of selenium, Se-78, Centrum Silver or tellurium-125, X, on the instrument response, Y, in minimizing the variability of the X-intercept around 0.5. In this talk, we derive an approximation of this variance. The formula has a complicated form that depends on the poulation intercept, the slope and the X-values.

# Victoria Ellison, Allison Fusco, and Jefferson Livermon, James Madison University

# Applied Multivariate Statistical Methods for Assessing and Improving Health Care Systems (COMAP Presentation)

Using exploratory factor analysis, we compute a method for generating a national "health score" that will effectively assess the quality and efficiency of a nation's health care system. We accomplish this by clustering and assigning weights to different metrics that can be used to compute a score measure the health status. With this method, we calculate the health scores of Sweden, Mexico, the United States, and Canada in each of our three areas of Childhood Healthcare, Adult Healthcare, and Mortality. We then developed multiple regression models for each of the eight metrics that comprise our factor scores. With these models we objectively analyze the strengths and weaknesses of the four countries' health care systems and are able to make specific recommendations for improvement. We restructured the health care system of Canada starting with the individual factor scores and utilize our models to asses the effectiveness in various changes in terms of the magnitude of change on our metrics and the factor scores.

# Douglas Fordham, James Madison University

# Graphing Goldbach

The Goldbach Conjecture has gone unproven for over hundreds of years. Now with the advancement of technology, we are able to better study the conjecture and collect data that before would have taken years to attain. Using this new data set, we can create different graphs that could potentially lead to an answer for the conjecture.

Deena Hannoun and Leslie Hindman, James Madison University

# Optimal Energy Allocation Strategy for Multiple Constrained Resources with Overlapping Generations

Our model focuses on the optimum energy allocation strategy for two subpopulations, *X* and *Y*, in a competitive environment. The two subpopulations must allocate energy between searching for two resources necessary for reproductive success: food and shelter. We use difference equations to model the population in time using overlapping generations. Individual fecundity is based on the amount of food obtained per individual, and the probability of successful reproduction depends on finding shelter. Previous work addressed the case of non-overlapping generations. We implement overlapping generations by including a fraction of the population that survives into the next generation. For given strategies and survival probabilities, we determine which population survives and which goes extinct.

## Elizabeth Hargraves, St. Mary's College of Maryland

#### LC Loops and Quasigroups

A quasigroup is a set along with a closed binary operation that has unique inverses but is not necessarily associative, and a loop is a quasigroup with an identity element. To better understand the structure of quasigroups, we consider identities that are weaker than the associative law, called Bol-Moufang identities, such as ((xy)x)z=x(y(xz)). In this research project, we consider loops and quasigroups that satisfy the Bol-Moufang identity (xx)(yz) = (x(xy))z. We explore certain subsets of these loops and quasigroups generated using the other Bol-Moufang identities, determine if they are subquasigroups, test if they are normal, and explore the structure of their quotients.

# Kaitlin Long, Randolph-Macon College

# Child's Play: A Brief History of Fermat's Last Theorem

This little theorem is intelligible to children, but took hundreds of years to solve. The man who ultimately published the accepted proof first became interested in the problem in grade school.

## Dan Simonsan, James Madison University

# n-Paper-Scissors: Rock-Paper Scissors as Vectors

Rock-paper-scissors is a valuable game that could be used to simulate real-life phenomena. It can be represented as a set of column vectors. Such representation allows the number of options to increase to any size yet maintain the balance of the three-option game. Computer representation can be used to further explore the properties of a set of rock-paper-scissors vectors.

# Andrea Sims, Virginia State University

# Bernstein Inequality for subclasses of Polynomials of Degree 2

Let  $P_n$  denote the class of all algebraic polynomials of degree at most *n* of a complex variable. It is well known from *Bernstein's Theorem* that on a unit disc, the maximum of the modulus of the derivative of a polynomial from  $P_n$  is less than or equal to *n* times the modulus of the polynomial. Equality holds if and only if the polynomial is  $p(z) = cz^n$ , where *c* is a constant. Using basic results from Complex Analysis and knowledge of Trigonometry and Calculus, we will give proof of many subclasses of  $P_2$ .

#### Brandon Strawn, James Madison University

# Rhetoric and Mathematics: Reviving the Exploration of a Transdisciplinary Bridge

In my unique perspective as an undergraduate student of rhetoric and mathematics, I have stumbled upon a secret: these two subjects, often billed as two of the most polarized realms of academia, are not as mutually exclusive as many claim them to be. Following this impulse, I learned that while their antithetical nature is often argued outside both disciplines, most rhetoricians and mathematicians are not surprised by the comparison. Furthermore, there has been some research in the area, the most noteworthy of which, though, was more than twenty years ago. Determined to correct this misconception, renew the study of their comparison, and present a more accurate general portrayal of both subjects for the laypeople, I set about to rebuild the bridge between rhetoric and mathematics. I learned that rhetoric can be studied and utilized mathematically both correctly and incorrectly, and that mathematics is rhetorical not only with regards to the communication thereof, but deep within its foundation as well. My research is useful not only to rhetoricians and mathematicians, but to the world

outside of both as well, illustrative of the understanding and progress such academic transdisciplinarianism can benefit.

# Jamey Szalay, James Madison University

# Applied Cryptography and Steganography

Ever wondered how to hide information on a computer? This talk will cover the concept of scrambling (Cryptography) and hiding (Steganography) data by embedding it into a picture file. Additionally, JBDMCOHUWRXRKSDLRKTRXGDENQYIMJUBQGMEEDIB. Find out what this means at the talk.

## Charles Tannouri, Towson University

# Game Theory: Enumerating Payoffs to Determine the Optimal Course of Action Within Role Playing Video Games

In modern video gaming, the genre of Role Playing Games have grown to be wildly popular. It can be demonstrated that these games follow the basic predicted structures of Game Theory. However, the question arises how to numerically weigh payoffs to determine a player's optimal course of action among all available strategies defined by the game. A generic model of a Role Playing Game will be analyzed per the principles of Game Theory and exemplified to demonstrate methods for determining such optimal strategies.

# Amy Winslow, Randolph-Macon College

# Modular Origami and the Trefoil Knot

Thomas Hull invented the origami module in order to make mathematical structures involving pentagons and hexagons. I will explain how I used these units to make a trefoil knot tessellated by polygons. I will show how to use graph theory to explore the possibility of creating a proper 3-coloring for this structure.

# Mervin Woodlin, Virginia State University

# Bernstein Inequality for Self- Reciprocal Polynomials of Degree 2

Let  $P_n$  denote the class of all algebraic polynomials of degree at most *n* of a complex variable. It is well known from *Bernstein's Theorem* that on a unit disc, the maximum of the modulus of a polynomial from  $P_n$ is less than or equal to *n* times the modulus of the polynomial. Equality holds if and only if the polynomial is  $p(z) = cz^n$ , where *c* is a constant. A polynomial  $p(z) = a_0 + a_1z + a_2z^2 + ... + a_nz^n$  in  $P_n$  is called self reciprocal if  $a_v = a_{n-v}$  for  $0 \le v \le n$ . The question is if *p* is self reciprocal, what is the best estimate of the maximum of the modulus of the derivative of a polynomial from  $P_n$  in terms of the modulus of the polynomial? Using basic results from Complex Analysis and knowledge of Trigonometry and Calculus, we will find the sharp estimate of the maximum of the modulus of the derivative of a polynomial from  $P_n$  in terms of the modulus of the polynomial on a unit disc. Some properties of this class will also be discussed.

# MD-DC-VA Section MAA Spring 2008 Meeting at EMU and JMU

# **Student Poster Abstracts**

Dan Boxer, Brent Kohler, and Molly McHarg, James Madison University *In Your Face!* 

Our goal is to create a program using matrix operations in Matlab that identifies a person's face despite different facial expressions.

Christopher Boyls-White ,Virginia Military Institute

# Barcodes

Undoubtedly everyone has seen a barcode. They have changed the commercial scene, by saving time and money due to their expediency. Barcodes have provided a median for automatic rapid data input into the computer. This research project concentrates on the mathematics involved with barcodes. We will briefly review the history of barcodes, investigate features of 1-dimensional (specifically the UPC and the POSTNET) and 2-dimensional. Then we will focus on the PDF417 stacked barcode. In this part we will discuss in detail the encoding and decoding algorithms, error detection and correction involvement, image capturing methods for converting barcode graphics into computer data, and our MATLAB program algorithm for coding a users desired data. With our ability to reengineer the PDF417 code with MATLAB, we can now create our own version of a stacked barcode with the possibility to develop a unique barcode. We expect the potential for customization capabilities for specific visual appealing for a user while still ensuring computer readability.

# Elizabeth Clawson, Caleb Reed, and Ben Shuey, James Madison University

# Markov Chains in a Financial Setting

The poster is an exploration in the use of a Markov chain to model financial transactions between companies.

# Mike Dankwa and Sandi Van Dolson, James Madison University

# To Multiply, or Not to Multiply: A Tale of Upper Triangular Matrices

Solving upper triangular matrices with a shift variable, using different mathematical methods.

# Sean DeWyngaert, James Madison University

# Markov Chains and their effects on financial transactions

I will display how Markov Chains and Stochastic Matrices will model the financial transactions of several companies. In order to display this I will be using Matlab as the backbone to which my work will be based off of.

# Andrew M. Elgert and Jonathan Spiker, James Madison University

# Dissecting the Google PageRank Algorithm

Google became a premier search engine in the mid 1990s by returning far more relevant search hits than any previous search engine. We analyze the basic algorithm behind Google's success, the PageRank algorithm, which uses linear algebra processes to return relevant hits.

# Lynn Grubb and Marianne Heldmann, James Madison University *Matrix Mania*

Analysis of different methods of solving shifted triangular systems.

# Mesbaul Haque, Evan Jacobs, and Sean Lawrence, James Madison University *Exploring Google's Page Rank Algorithm*

Explains the basic algorithm by which Google ranks webpages.

# Karen Hesson, Salisbury University *Curves in Clay*

This poster will outline some of the work I have been doing on sculpting ascetically pleasing surfaces. I have been studying some of the geometrical properties of these surfaces, such as the mean and Gaussian curvatures, and then attempted to make them come to life in a body of clay. The majority of this work has concentrated on minimal surfaces, such as the Catenoid and Enneper's Surface. I will outline the mathematical properties of these surfaces and other ascetically pleasing surfaces as well as the techniques I used to sculpt them.

# Jennifer La Nini and Yanitsa Staleva, James Madison University

# Google's PageRank Algorithm

The poster will explain part of Google's Page Rank algorithm, which assigns an importance ranking to each page on the web, allowing Google to rank the pages and present the user more important pages first. Understanding how to calculate PageRank is essential for anyone designing a web page that they want people to access frequently.

# Katherine Maginn and Lane O'Brien, James Madison University

# Ranked from One to Google

Our project seeks to analyze Google's page-ranking algorithm, a program which determines the importance of each webpage based on the respective ranks of the pages which link to that page. This technique enables Google to place more popular websites on its earlier pages and present them to the user first. The purpose of this project is to study this algorithm and gauge its accuracy.

# Chase Maitland and Jessica Rohrer, James Madison University Collision of Math and Finance

Using math and computer software to help solve a Markov process.

# Ryan Mccormack and Brian Scott, James Madison University

# Stochastic Matrices and Markov Processes

Presentation on the application of Markov processes, stochastic matrices, eigenvalues, and the Power Method to Finance.

# Stephen McGinley, Emmett Randel, and Robert Turner, James Madison University *Eigenface, you can face, we all can face*

A database of pictures of faces is made and the eigenface algorithm is performed on them.

# Ronald Pandolfi, Randolph-Macon College

# Doubting Dodgson's Method of Determinants

The mathematical studies of Charles Lutwidge Dodgson -- most well known for his fiction-writing under his alias "Lewis Carroll" -- included a method of calculating determinants. Dodgson's method of determinants is easy to learn and works great for commonly encountered matrices; however it can have complications. It can be shown that Dodgson's method can be unstable and produce large error when using floating point arithmetic operations.

# Noah Scribner, Virginia Military Institute

# Early History of the Department of Mathematics at VMI

Since the time of its founding in 1839 the study of mathematics has been of great importance to the cadets of The Virginia Military Institute (VMI). However, there is no formal recorded History of

the Department of Mathematics at VMI. In this poster session we will take an in-depth look at the Mathematics Department from 1839 thru 1899 to find out who the faculty were, what subjects were taught, and what texts were used.

# Danielle Shiley, Roanoke College

# Applying Calculus to the AIDS Epidemic in Cuba

This project seeks to employ a differential equations model that best represents real data on AIDS in Cuba from 1986 to 2000. Based on work by previous researchers, this model categorizes people as being

healthy but susceptible to HIV+ and AIDS, people who have contracted HIV+ (and could later develop AIDS), and people suffering from AIDS. The equations describe how susceptible and HIV+ people move into each category of AIDS based on their contact with the rest of the population. Parameters representing birth and death rates, the rate at which HIV+ is transferred, the rate at which AIDS is developed, and the death rate due to AIDS are estimated based on data collected in Cuba from 1986 to 2000. Error for these estimations as well as stability of solutions is also analyzed.

## Matthew Spencer, James Madison University

#### Shifted Upper Triangular Systems

Efficient algorithms for solving the system  $(S^T-lambda^I)x=b$ , where *b* is an *nx*1 vector, *S* and *T* are *nxn* upper triangular matrices, lambda is a scalar (the shift), *I* is the *nxn* identity matrix and *x* is an *nx*1 vector of unknown components.

# Teri Swinson, Lok-Kun Tsui, and Glenn Young

## Physics Applications of Trigonometric Interpolation

With the use of MATLAB, we apply the method of interpolation to physics data. Polynomial interpolation does not work well on periodic data, so we use trigonometric functions to interpolate the data points. Specifically we apply this to solar irradiance data from NOAA.

# Cadet Michael Werle, Virginia Military Institute

# Field Artillery and Geometric Overlap

Through the employment of a wide variety of weapon systems, field artillery is capable of causing more death and destruction on the battlefield than any other force in an army's arsenal. These weapons rely on many branches of mathematics in both their design and employment. In particular, the simultaneous explosions of multiple shells on a single target can be examined in greater detail using various mathematical techniques to measure geometric overlap. During the course of this project the overlaps of increasingly complex shapes beginning with the square and concluding with the sphere were examined and calculated. Several parameters were considered including object translation and 3D rotation. In the final stages, we programmed these simulations using MATLAB and concluded by running these programs to measure both the overlap and total volume of up to five spheres randomly offset from a common center in order to simulate what kind of total and concentrated artillery coverage might be expected on a target from a barrage of airburst artillery rounds.