Optimal Shut the Box

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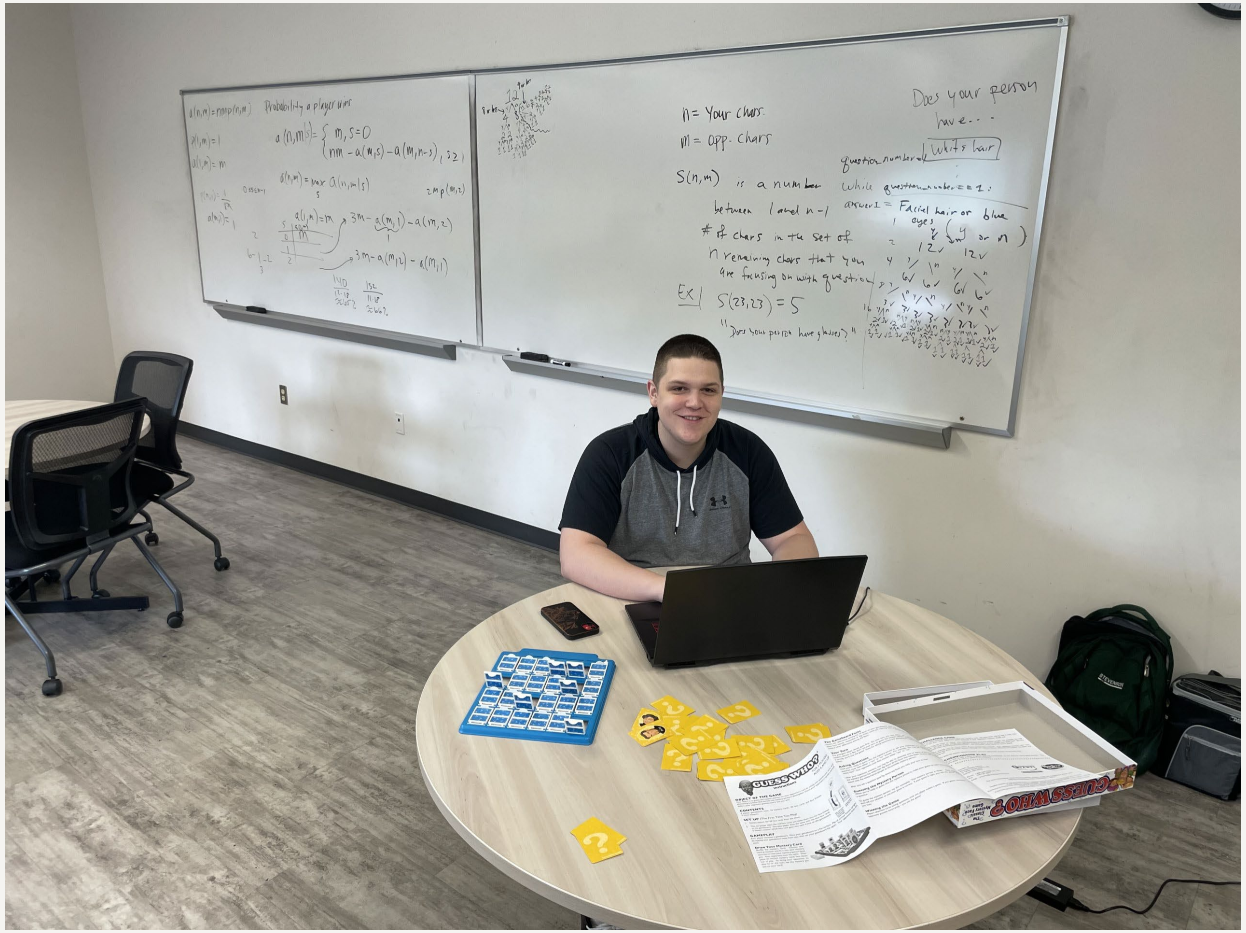
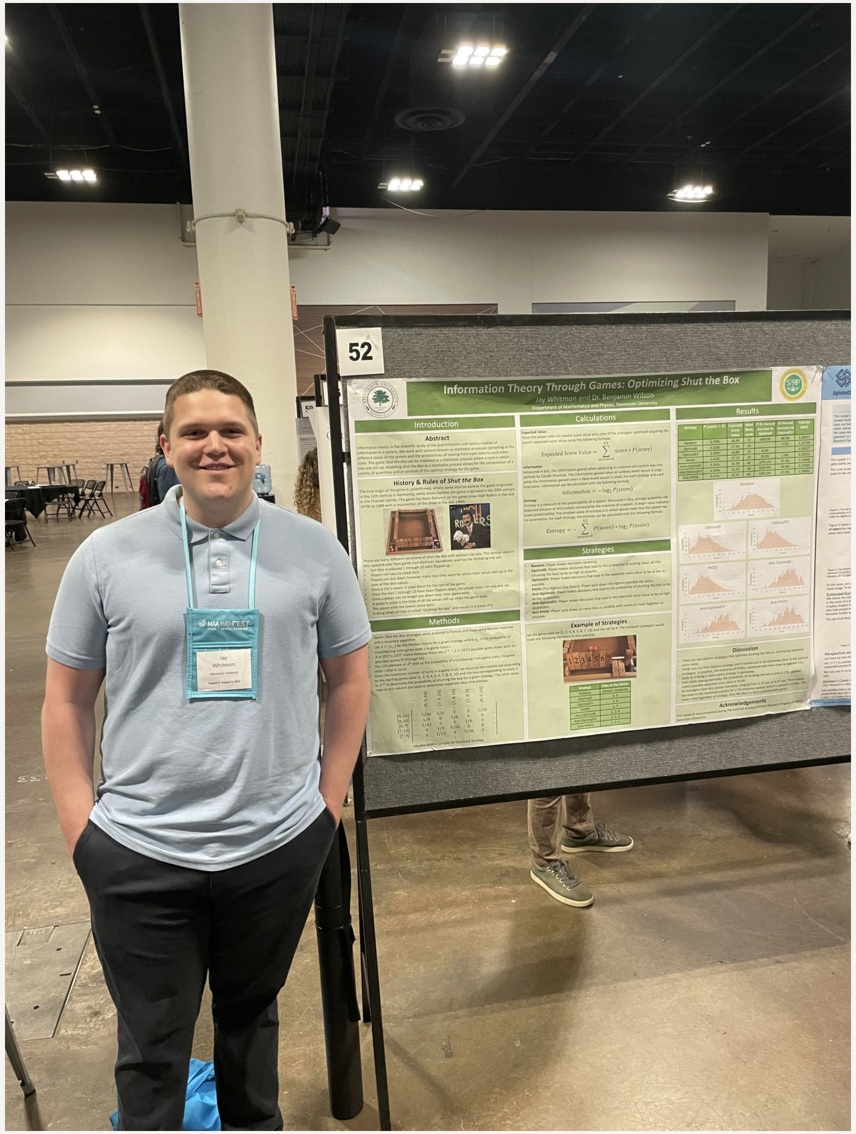
Engineering, Math, and Physics

This talk is based on work conducted with Stevenson Applied Mathematics major Jay Whitmon ’24.





Shut the Box

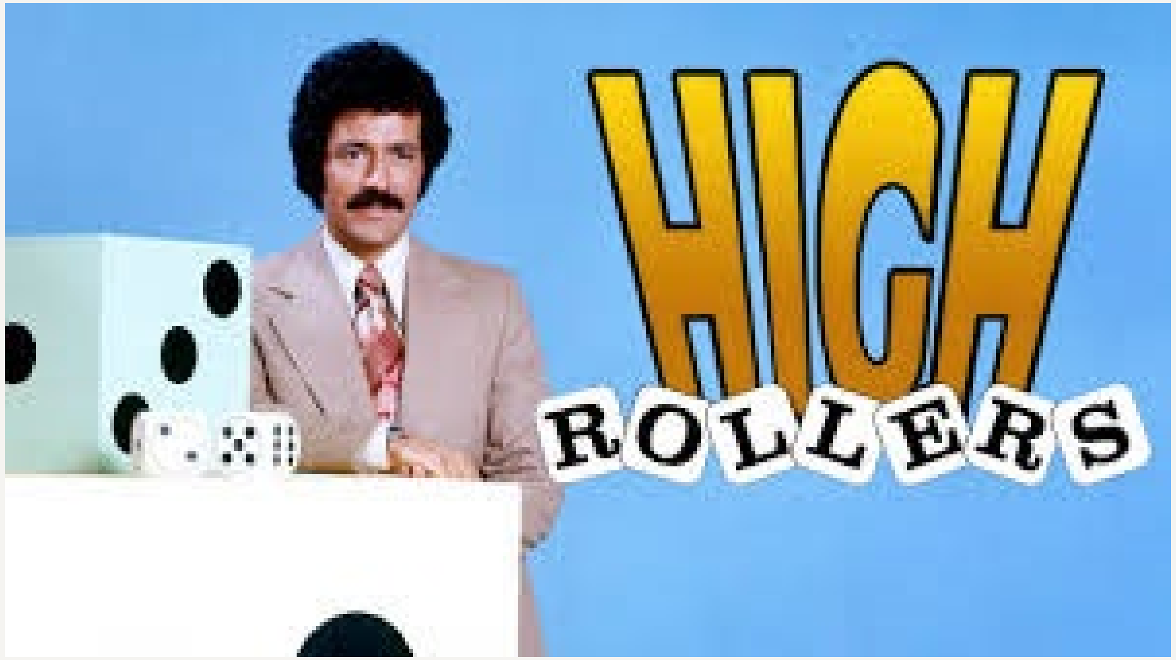
Shut the Box is a dice game where players begin with tiles numbered 1 through 10 (9 and 12 variations exist as well) standing upright. On each turn, a player rolls two dice (or one die in some variations) and then “shuts” any combination of available tiles that add up to the roll total.

Basic Rules (“ApudArmis Ruleset”):

- Ten tiles numbered 1 through 10 begin face up.
- Players roll two six-sided dice.
- After each roll, players may flip down any combination of tiles whose values sum to the total shown on the dice.
- Once a tile is flipped down, it remains down for the rest of the game.
- When tiles 7 through 10 have all been flipped down, the player switches to rolling a single die.
- The game ends when the player can no longer flip down tiles that match the roll.
- The player’s score is the sum of all tiles left up at the end of the game.
- The lowest score wins.
- Flipping down all tiles is called “shutting the box” and results in a score of 0.

History of *Shut the Box*

- Unconfirmed histories of the game suggest a variety of origins, including 12th century Normandy (northern France) as well as the mid 20th century Channel Islands (Jersey and Guernsey), which one source credits to a man known as 'Chalky' Towbridge.
- A 1967 edition of Brewing Review describes the game as being native to the Channel Islands, and records it being played in Manchester pubs in the mid-1960s.
- Shut the box is the basis of the American television quiz show High Rollers, which ran from 1974 to 1976 and 1978 to 1980 on NBC with Alex Trebek as the host.



https://youtu.be/Qt9ZNqy-ZDM?si=QR7-gY4fsCkjKqAt&t=138

Let's play some *Shut the Box*!

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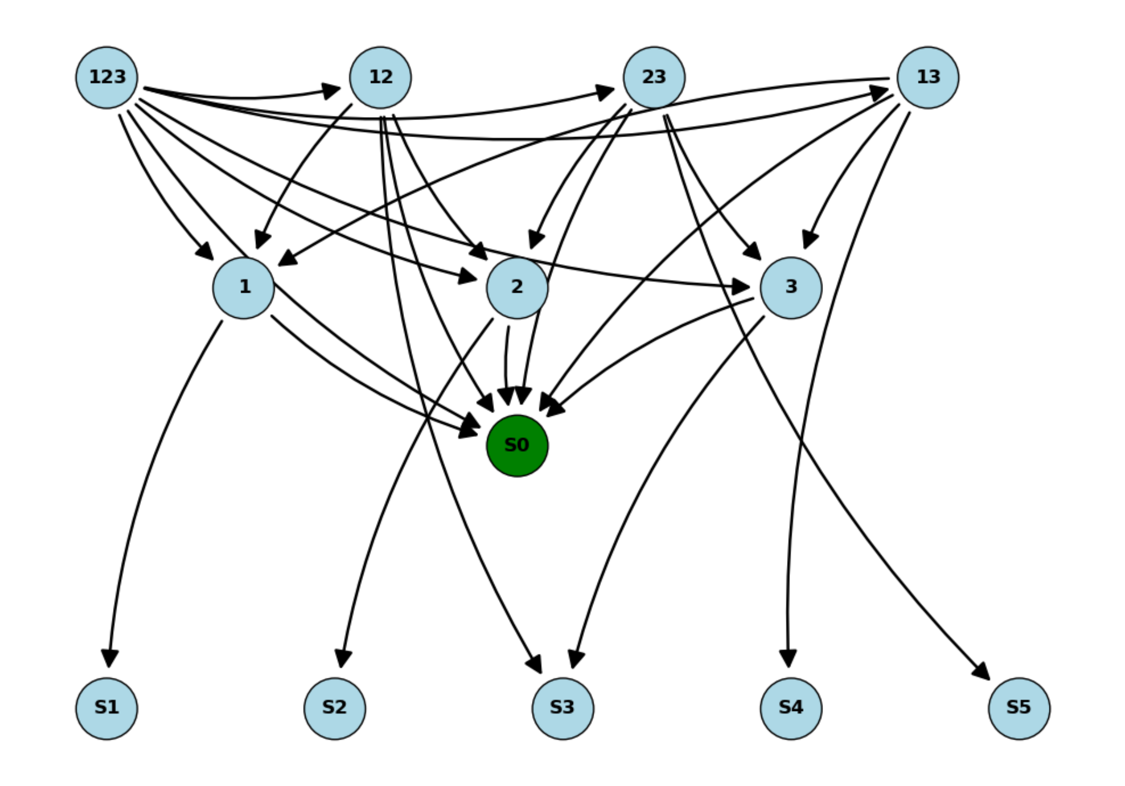
Optimal *Shut the Box*

By modeling *Shut the Box* as a stochastic process and using dynamic programming, we can determine the optimal strategy for every possible game state and dice roll.

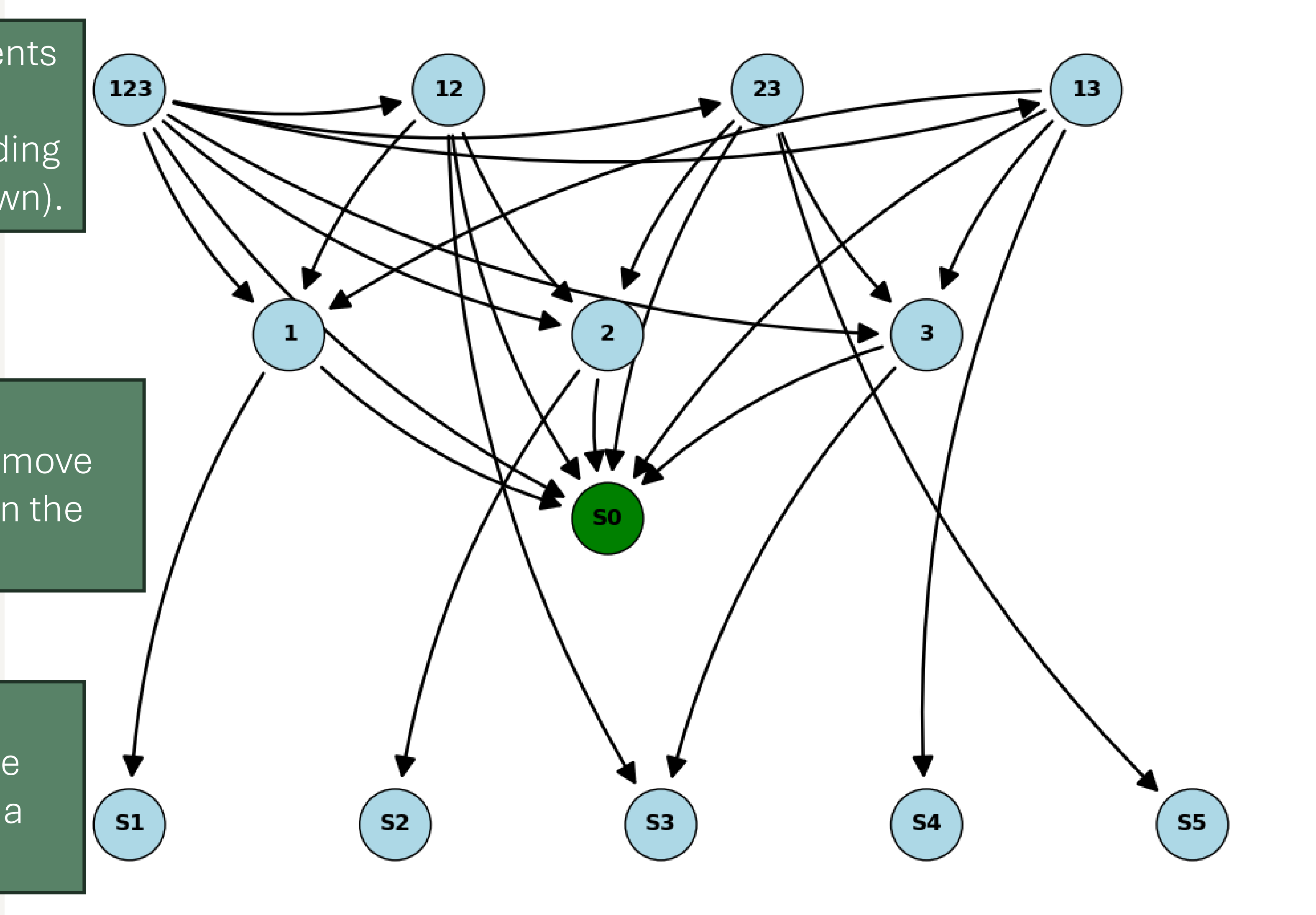
There are two possible parameters to optimize with potentially different strategies:

- Highest chance to shut the box
- Lowest possible score



Stochastic Process Model

Each node represents the set of tiles currently still standing (not yet flipped down).

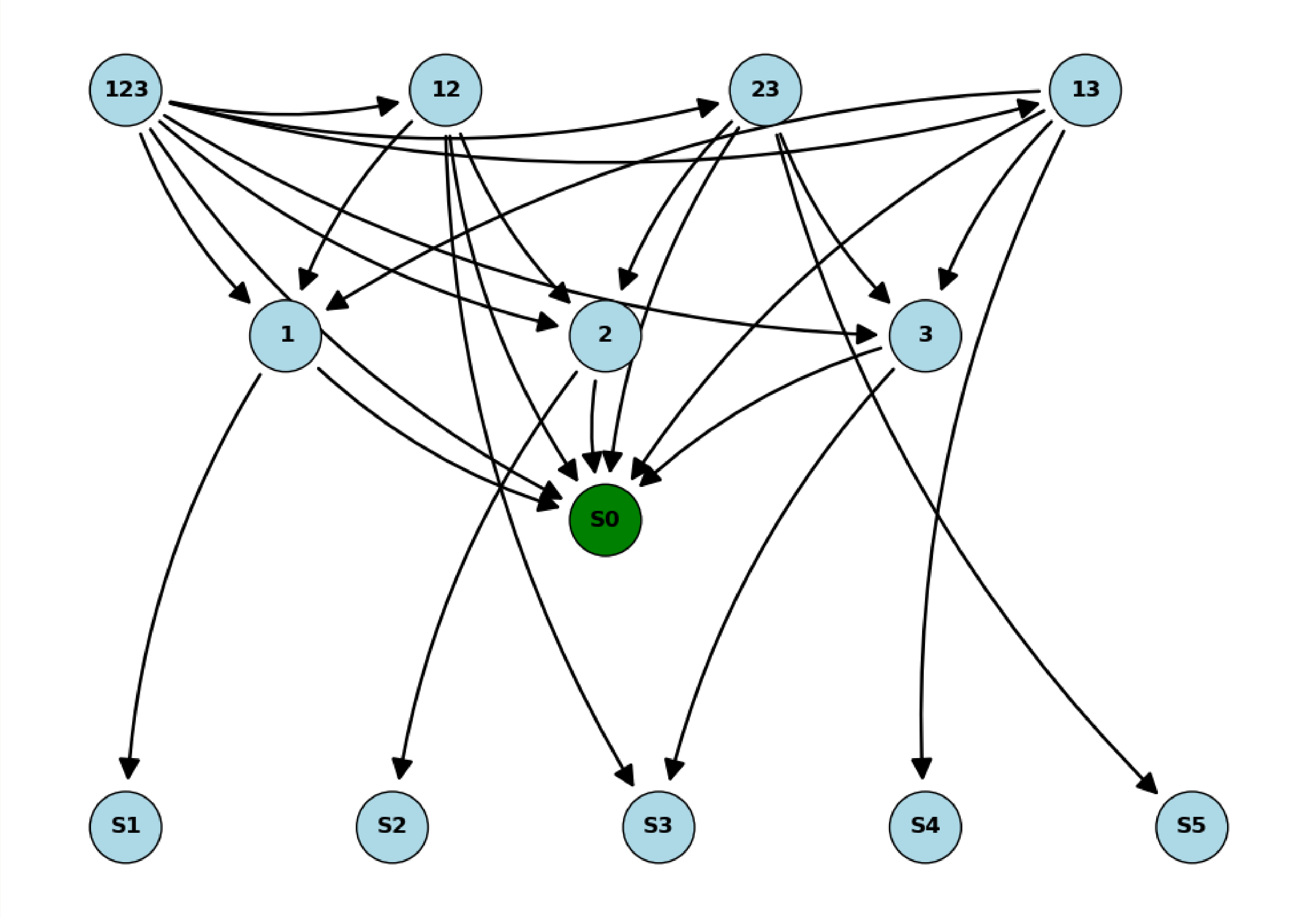


Each directed arrow represents a possible move the player can make on the next roll.

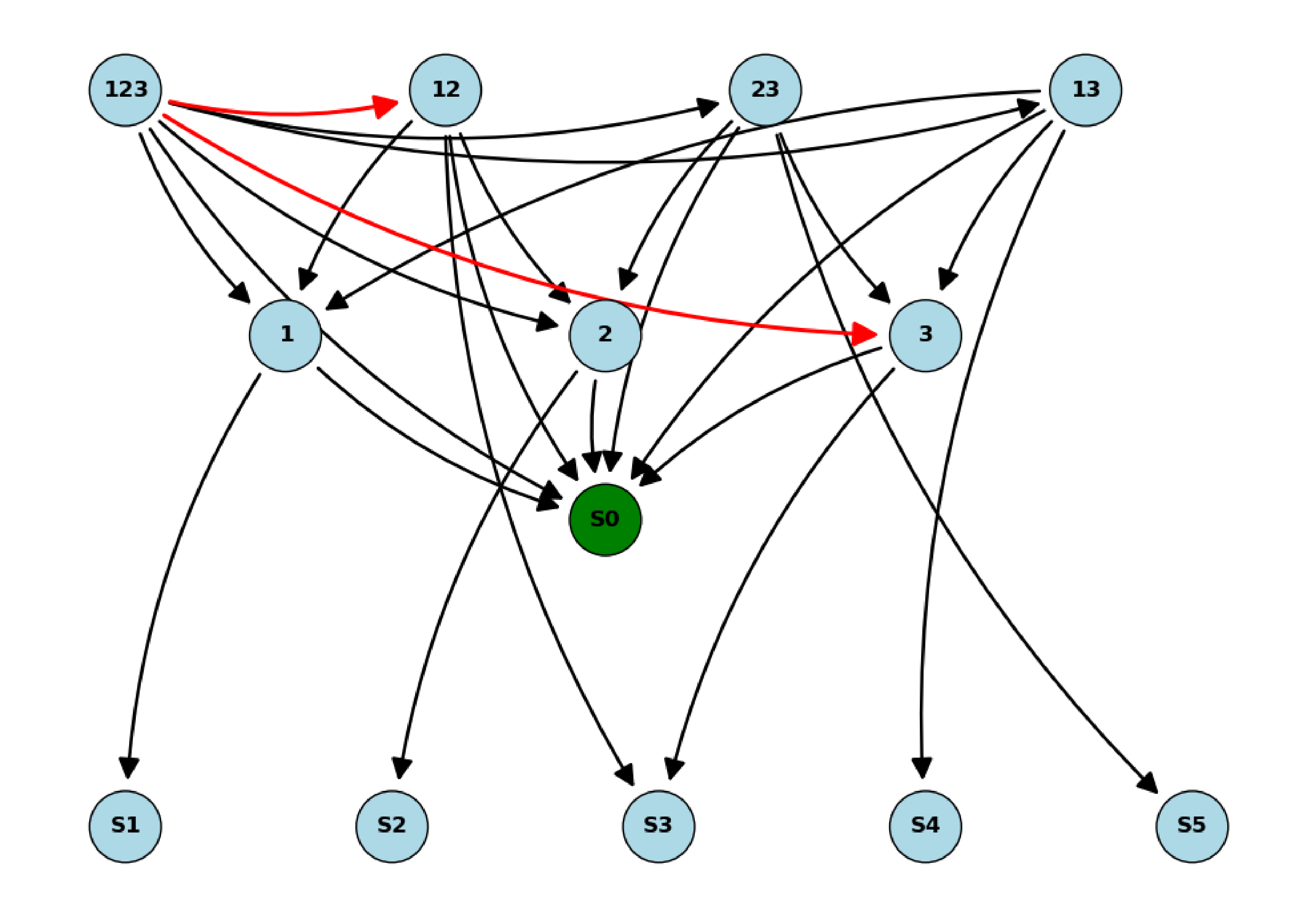
"S" nodes represent final scores when the player cannot make a valid move.

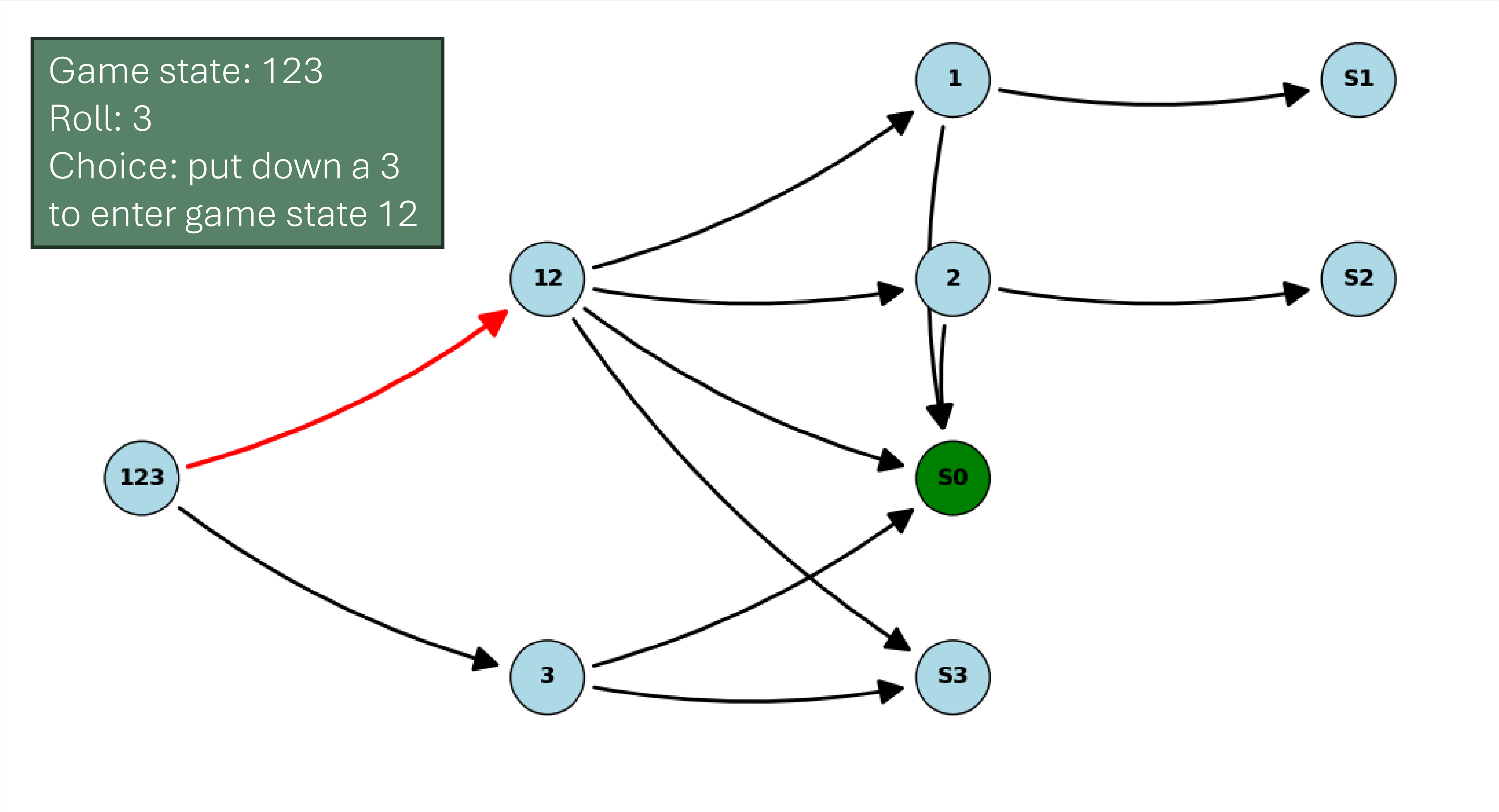
Stochastic Process Model

This model shows all possible game states starting from the position where tiles 1, 2, and 3 are up. It contains thirteen nodes. A complete model beginning with all 10 tiles up would contain 1,078 nodes.



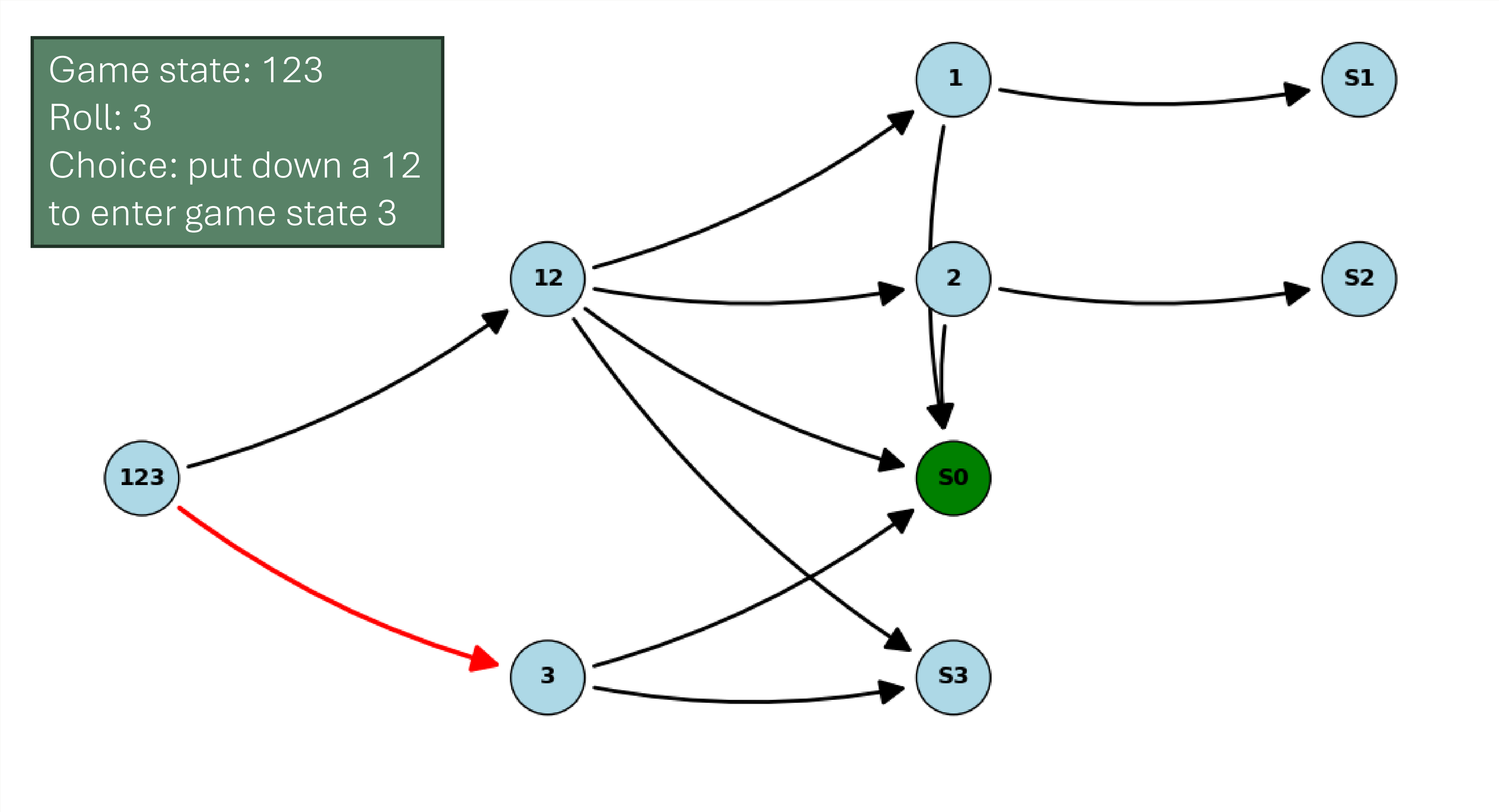
From certain game states and given certain rolls, a player must make a decision. For example, if you are in the game state 123 and you roll a 3, you can either put down the 3 and move to game state 12, or put down the 1 and 2 and move to game state 3.





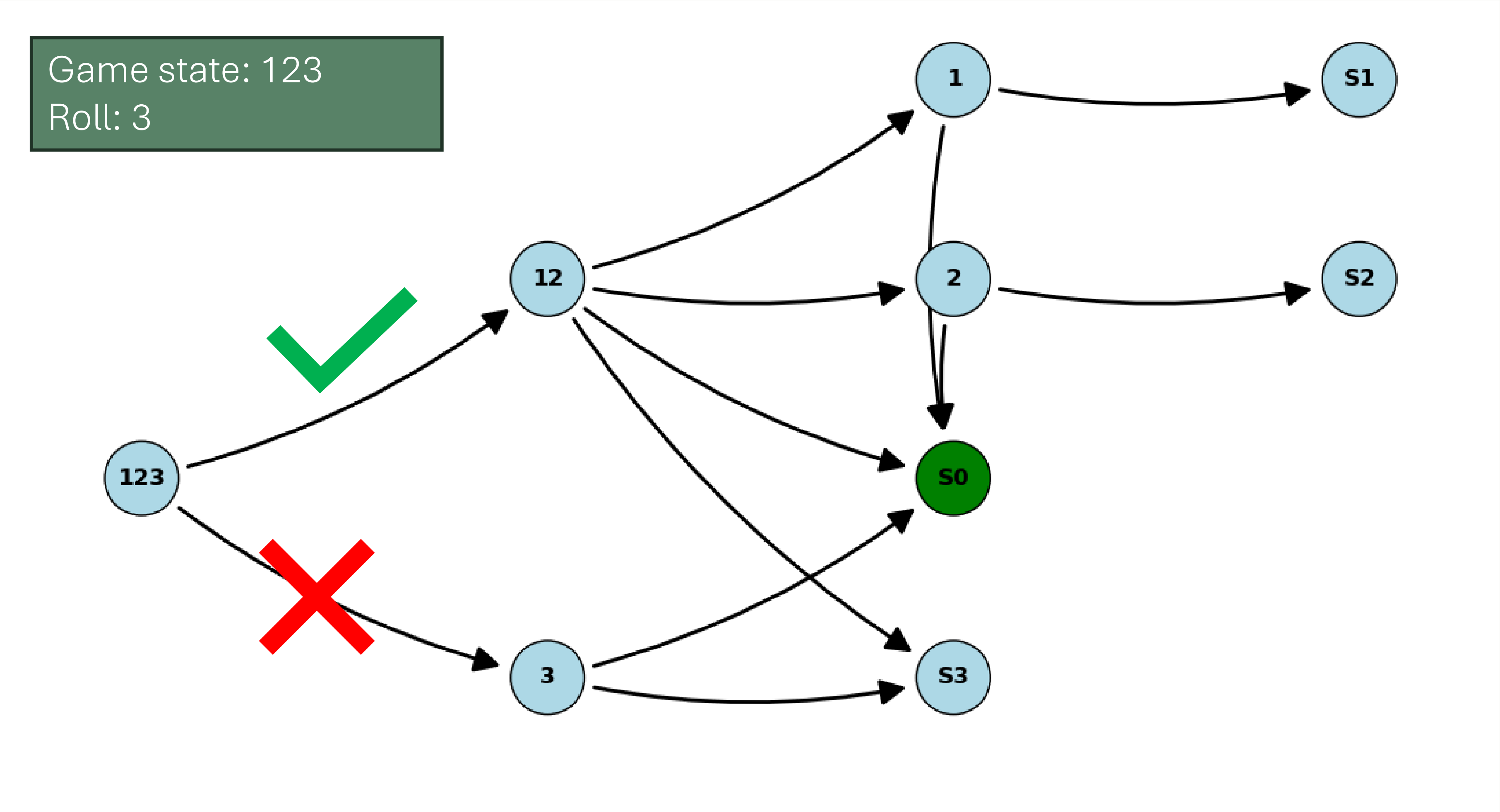
$$P(\mathrm{S0}) = \frac{1}{6} + 2\left(\frac{1}{6}\right)^2 \approx 0.22$$

$$\text{Expected Score} = \frac{1}{6} \cdot \frac{5}{6} \cdot 1 + \frac{1}{6} \cdot \frac{5}{6} \cdot 2 + \frac{1}{2} \cdot 3 \approx 1.92$$



$$P(\mathrm{S0}) = \frac{1}{6} \approx 0.17$$

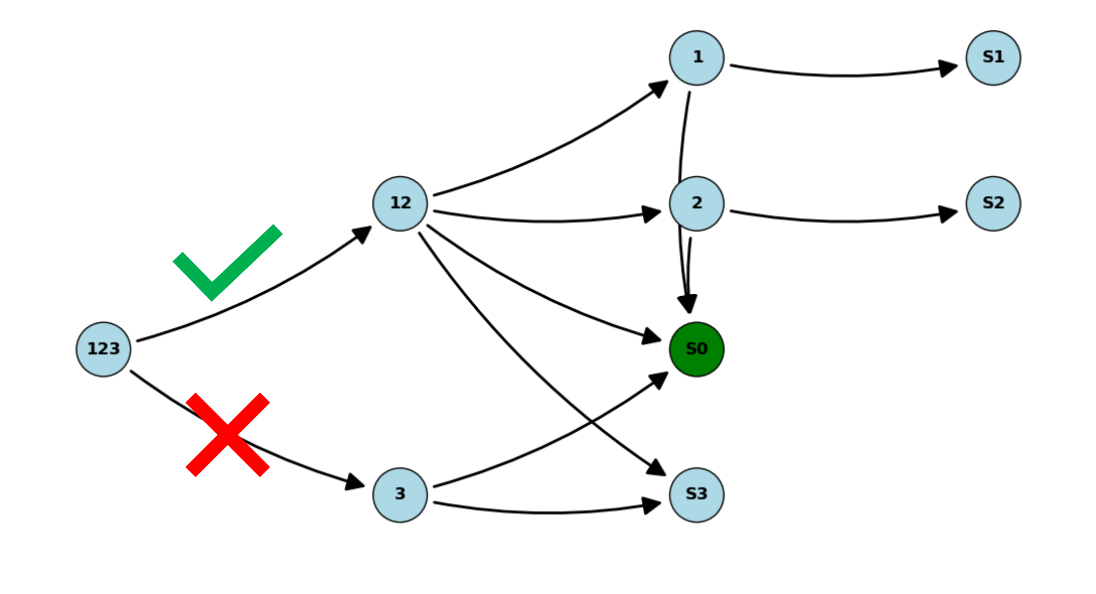
$$\text{Expected Score} = \frac{5}{6} \cdot 3 = 2.5$$



Whether we are trying to have the highest probability of shutting the box or lowest possible expected score, we should choose to put down the 3 tile in this scenario and enter game state 12.

Dynamic Programming for Optimal Shut the Box Strategy

- Dynamic programming provides a method for finding the optimal choice in every possible game state.
- We compute the best move in the smallest, simplest states first, then use those results to determine optimal decisions in increasingly larger states.
- For example: if you ever reach the state 123 and roll a 3, you should flip down 3 instead of 12 (to optimize for lowest expected score or probability of shutting the box).
- Once all optimal decisions are known for these small states, we move upward to states with more tiles remaining, using previously computed values to guide decisions until we reach the full starting state of the game.



Stochastic Matrix

- A stochastic matrix provides a way to keep track of every possible game state and how the game can evolve.
- Let $A = [a_{ij}]$ be the matrix where each row and each column represent a game state and a_{ij} is the probability of transitioning from state j to state i in a single turn.
- The matrix A^k gives the probability of moving from state j to state i in exactly k turns.
- This allows us to compute long-term behavior, expected outcomes, and the distribution of end states.

$$
\begin{array}{c}
\vdots \\
[9,10] \\
[8,10] \\
[8,9] \\
[7,10] \\
[7,9] \\
\vdots
\end{array}
\begin{bmatrix}
\cdots & [8,9,10] & [7,9,10] & [7,8,10] & [7,8,9] & [6,9,10] & \cdots \\
\cdots & \frac{5}{36} & \frac{1}{6} & 0 & 0 & \frac{5}{36} & \cdots \\
\cdots & \frac{1}{9} & 0 & \frac{1}{6} & 0 & 0 & \cdots \\
\cdots & \frac{1}{12} & 0 & 0 & \frac{1}{6} & 0 & \cdots \\
\cdots & 0 & \frac{1}{9} & \frac{5}{36} & 0 & 0 & \cdots \\
\cdots & 0 & \frac{1}{12} & 0 & \frac{5}{36} & 0 & \cdots \\
\cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{bmatrix}
$$

Strategies

- Random

  The player makes decisions completely at random.

- Optimal0

  The player chooses moves that *maximize the probability of shutting the box* (ending with score 0).

- OptimalEV

  The player chooses moves that *minimize the expected final score*.

- PHOD — Put Highest One Down

  The player always flips down the highest-valued tile available.

- Anti-Optimal0

  The player chooses moves that *minimize the probability of shutting the box*.

- Anti-OptimalEV

  The player chooses moves that *maximize the expected final score*.

- Anti-PHOD

  The player flips down *as many tiles as possible*, preferring tile combinations that are *close together in value*.

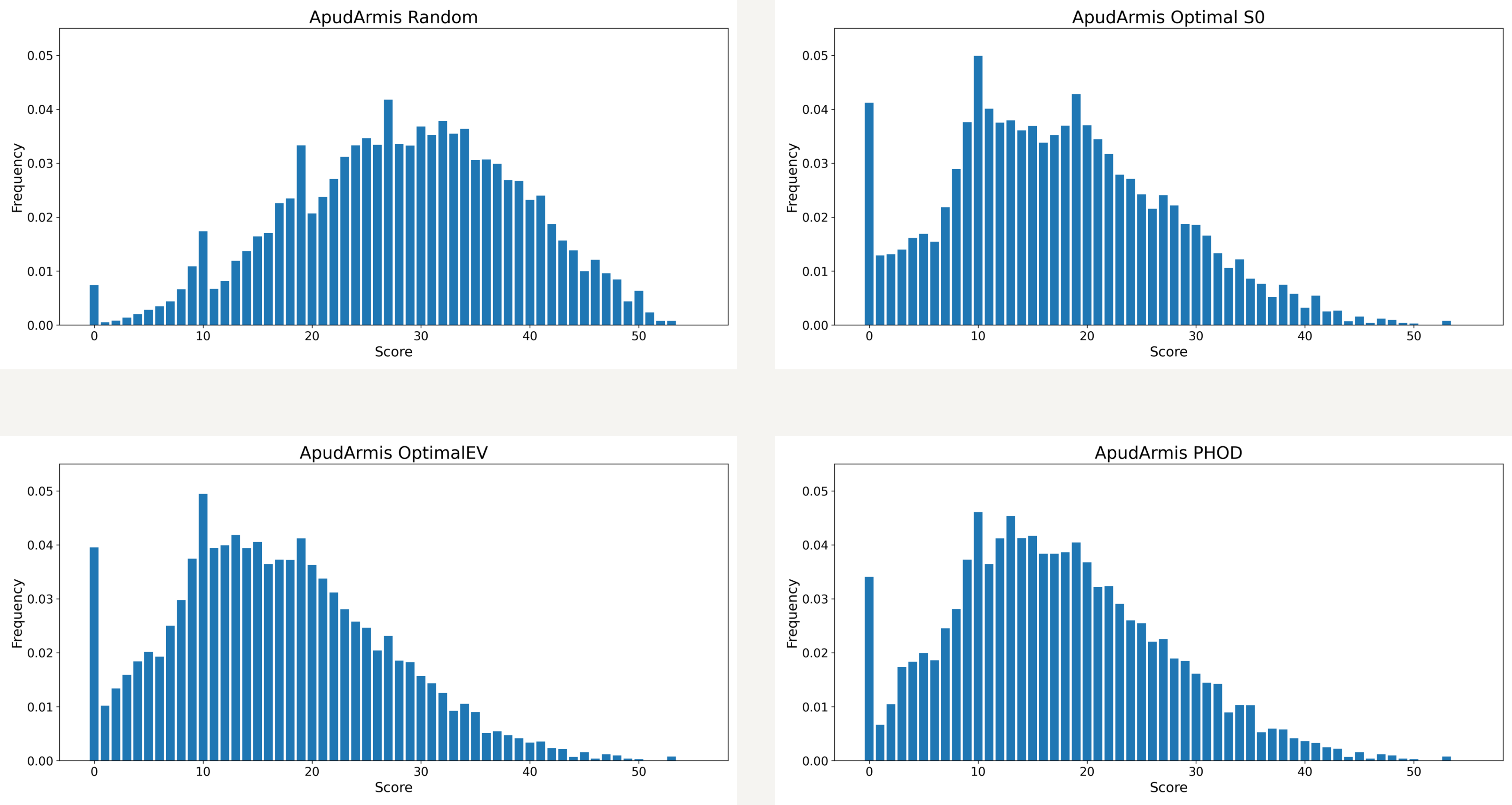
Example

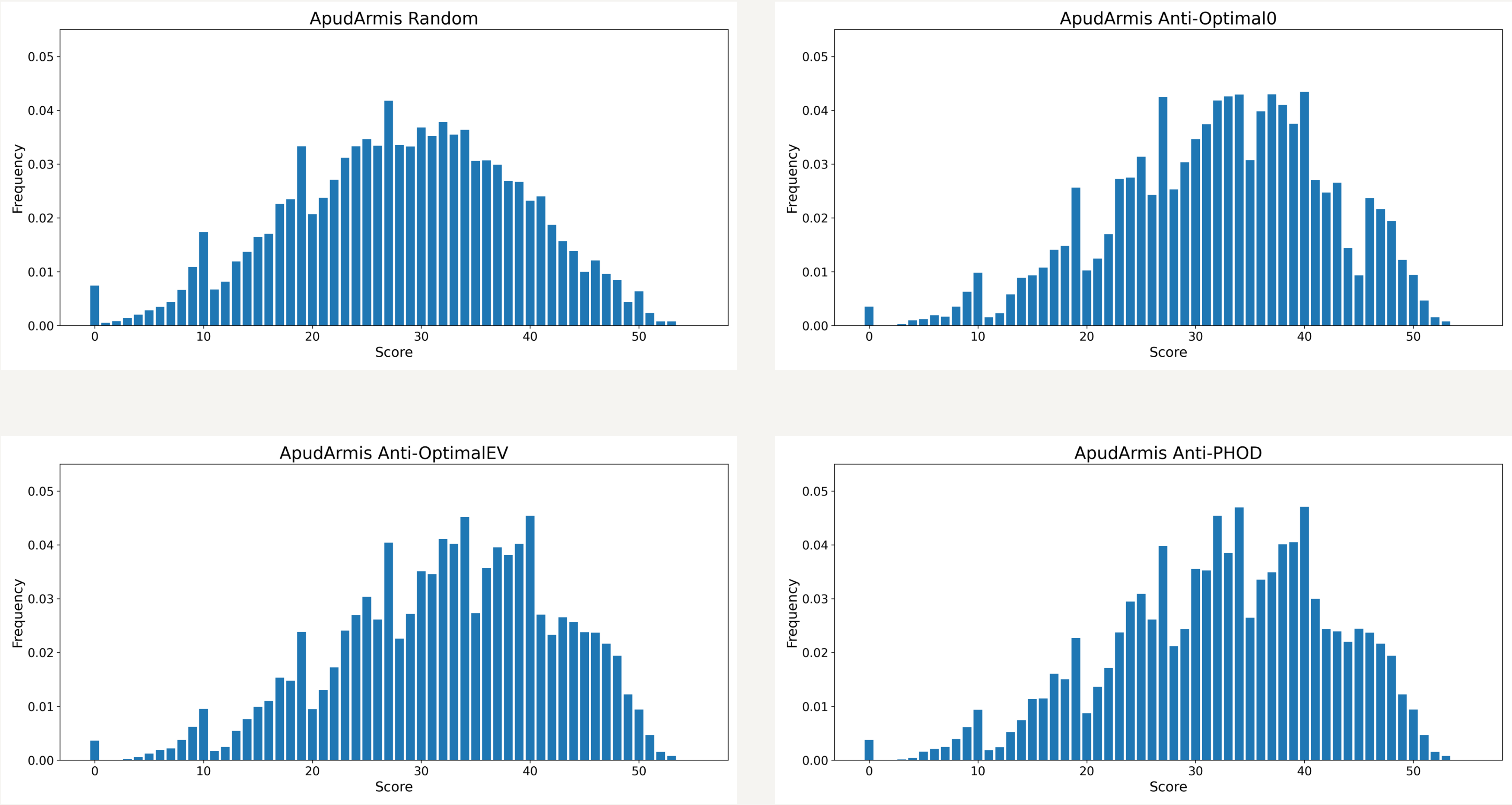
Game state: [1, 2, 3, 4, 5, 6, 7, 10]
Roll: 9

Strategy	Tiles to Put Down
Random	Random
Optimal0	4, 5
OptimalEV	3, 6
PHOD	2, 7
Anti-Optimal0	1, 2, 6
Anti-OptimalEV	2, 3, 4
Anti-PHOD	2, 3, 4

Results (ApudArmis Ruleset)

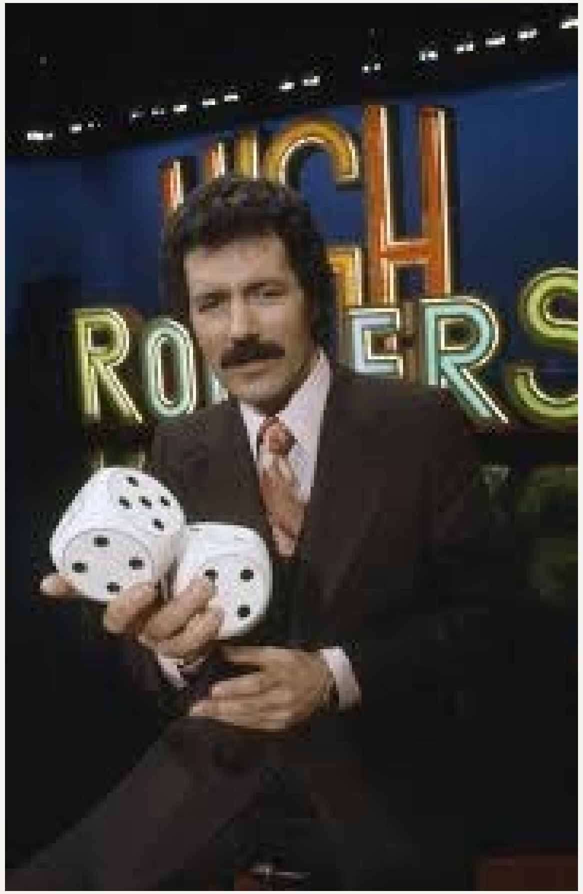
Strategy	Probability of Shutting the Box	Expected Value of Score	Most Likely Score
Random	0.74%	28.40	27
Optimal0	4.12%	17.44	10
OptimalEV	3.95%	16.89	10
PHOD	3.41%	17.23	10
Anti-Optimal0	0.35%	32.09	40
Anti-OptimalEV	0.36%	32.45	40
Anti-PHOD	0.37%	32.37	40

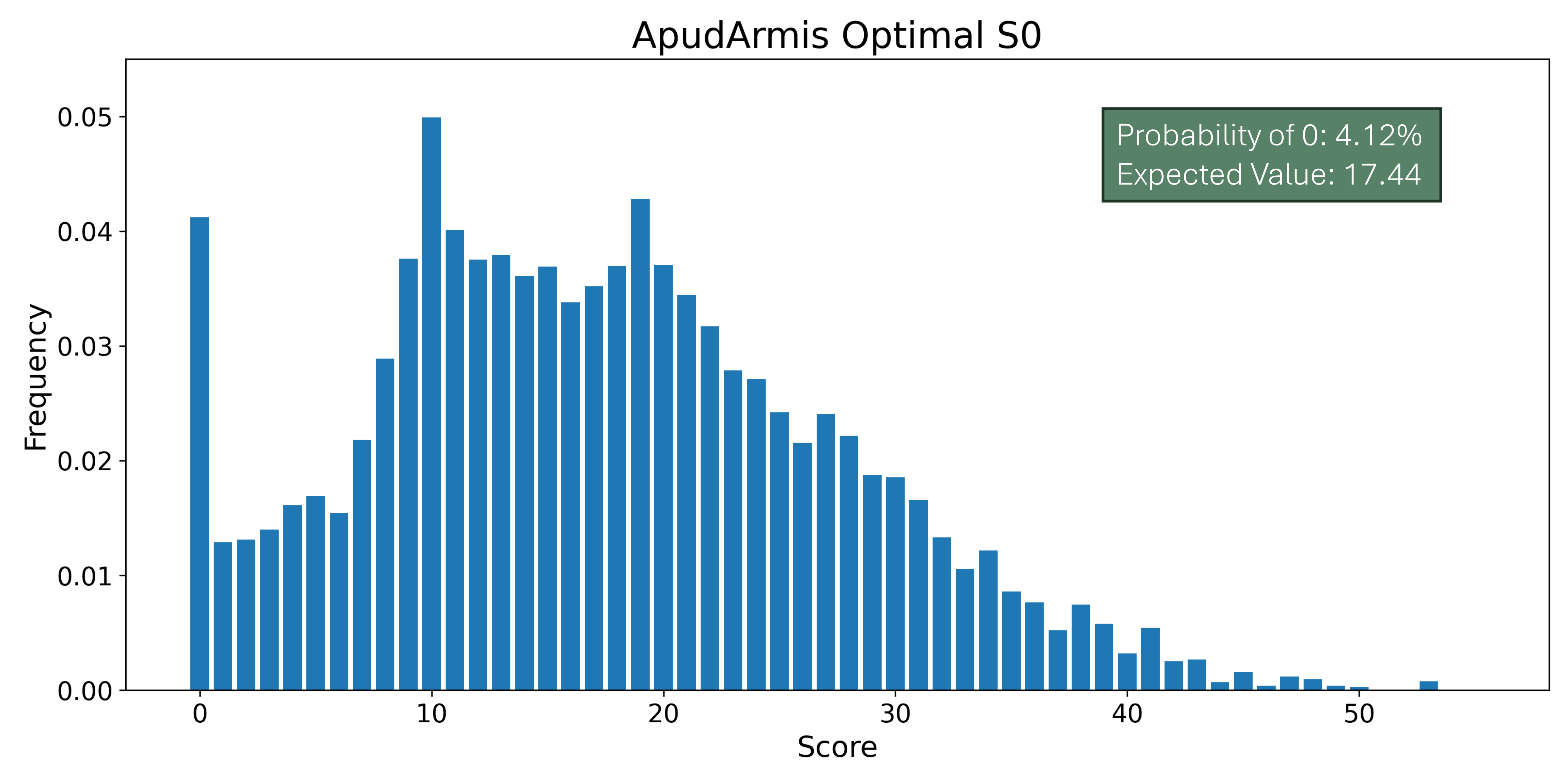

ApudArmis Random
ApudArmis Optimal S0
ApudArmis OptimalEV
ApudArmis PHOD
Frequency
Score
0.00
0.01
0.02
0.03
0.04
0.05
0
10
20
30
40
50

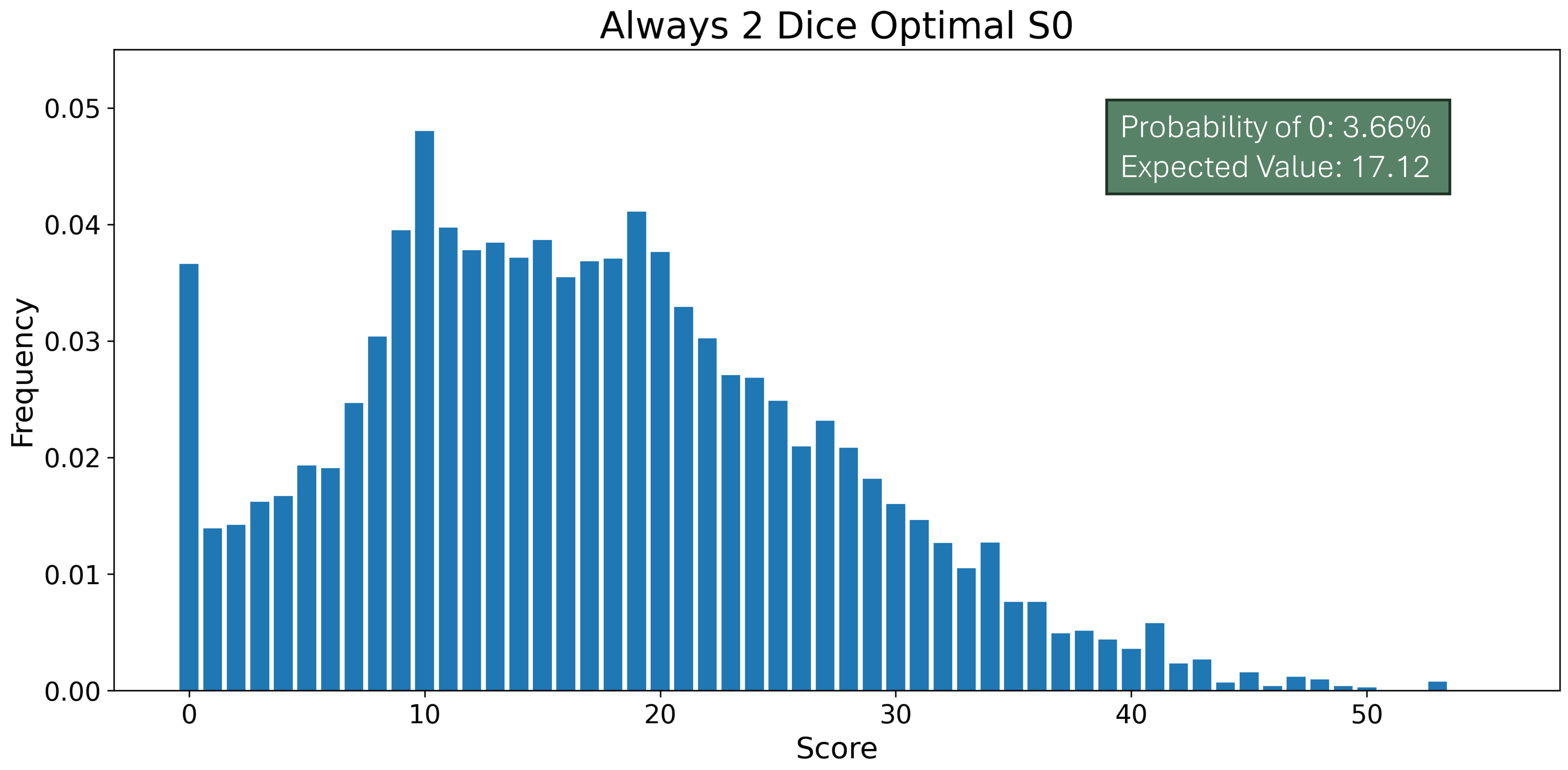

ApudArmis Random
ApudArmis Anti-Optimal0
ApudArmis Anti-OptimalEV
ApudArmis Anti-PHOD
Frequency
Score
0.00
0.01
0.02
0.03
0.04
0.05
0
10
20
30
40
50

Other Variations

The ApudArmis Ruleset require the player to use 1 die at a certain point. What if we always require the player to use 2 dice?




ApudArmis Optimal S0
Probability of 0: 4.12%
Expected Value: 17.44
Frequency
Score
0.00
0.01
0.02
0.03
0.04
0.05
0
10
20
30
40
50



Always 2 Dice Optimal S0
Probability of 0: 3.66%
Expected Value: 17.12
Frequency
0.00
0.01
0.02
0.03
0.04
0.05
Score
0
10
20
30
40
50

Discussion and Future Work

- There are two distinct strategies that optimize shutting the box vs. minimizing expected score value.
- PHOD turned out to be extremely close to the optimal strategies. In practice, the simplicity of PHOD, combined with how close to optimal it is, leads to it being a more useful strategy in gameplay.
- Even while playing optimally, the highest probability of shutting the box is only 4.12%, and the lowest possible expected score value is only 16.89.
- Future work includes analyzing other variations (different rules for the dice, different numbers of tiles, etc.)

Thank you!

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