Lord Rayleigh: A Quintessential Classical Applied Mathematician and Mathematical Physicist

Quintessential:

Representing the perfect example of something.

Brief introduction and career overview

Wikipedia Rayleigh

The Principle of Similitude

Hydrodynamic Stability Theorems

Rayleigh Scattering

Unrecognized Contributions to Scattering Theory

Lord Rayleigh: A Quintessential Classical Applied Mathematician and Mathematical Physicist

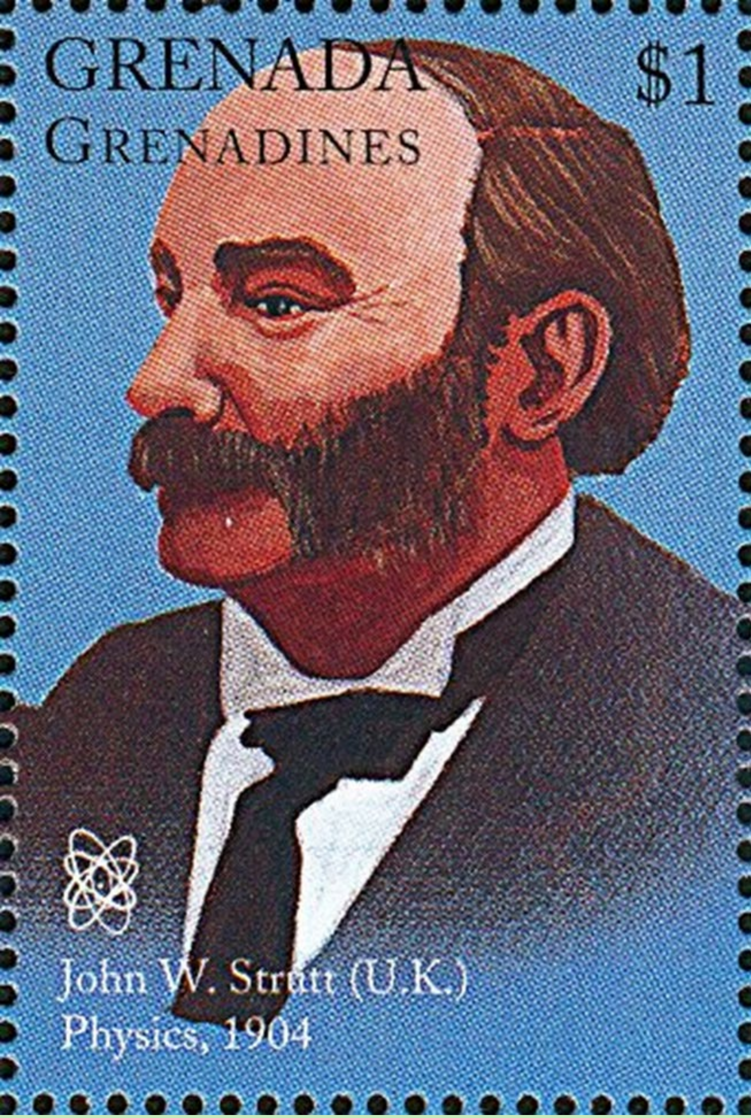
Born in Maldon, Essex, England in 1842; died in Witham, Essex in 1919)

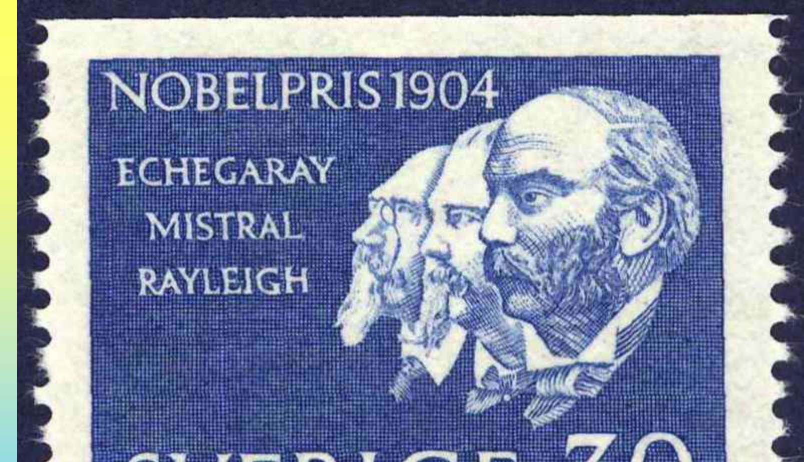
He was one of the very few members of "higher nobility" who won fame as an outstanding scientist.

A "selfie", taken around 1870.

Lord Rayleigh







Sweden 1964, Granada 1995,
Guinea-Bissau 2009

Overview

Otherwise known as John William Strutt, 3rd Baron Rayleigh, he made extensive contributions to science.

He spent his entire academic career at the University of Cambridge.

He served as President of the Royal Society of London from 1905 to 1908, and as chancellor of the University of Cambridge from 1908 to 1919.

Among many honors, he received the **1904 Nobel Prize in Physics** “for his investigations of the densities of the most important gases and for his discovery of argon in connection with these studies.”

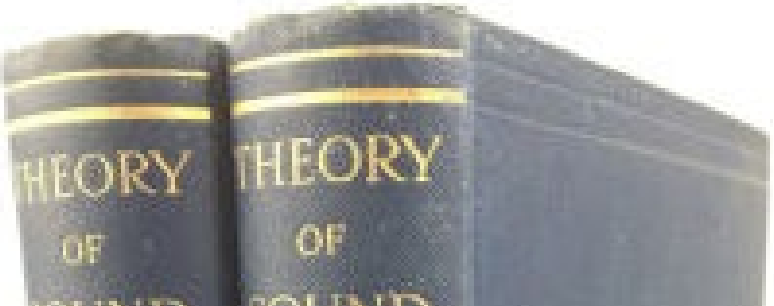
He was, therefore, also a superb experimental physicist.

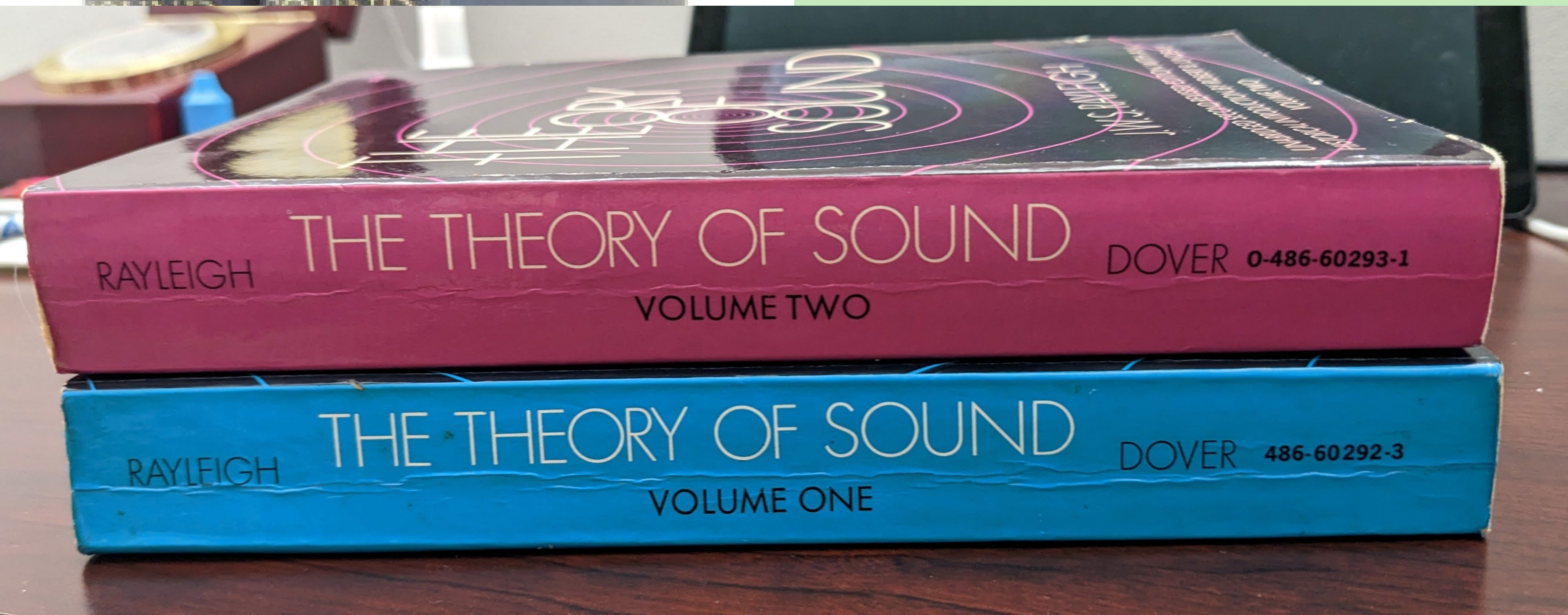
Lord Rayleigh's first researches were mainly mathematical, concerning optics and vibrating systems, but his later work ranged over almost the whole field of physics, covering sound, wave theory, color vision, electrodynamics, electromagnetism, light scattering, flow of liquids, hydrodynamics, density of gases, viscosity, capillarity, elasticity, and photography.

His patient and delicate experiments led to the establishment of the standards of resistance, current, and electromotive force; and his later work was concentrated on electric and magnetic problems.

Lord Rayleigh was an excellent instructor and, under his active supervision, a system of practical instruction in experimental physics was devised at Cambridge, developing from a class of five or six students to an advanced school of some seventy experimental physicists.

His *Theory of Sound* was published in two volumes during 1877-1878, and his other extensive studies are reported in his *Scientific Papers* – six volumes issued during 1889-1920. He has also contributed to the *Encyclopaedia Britannica.*




RAYLEIGH
THE THEORY OF SOUND
DOVER
0-486-60293-1
VOLUME TWO
RAYLEIGH
THE THEORY OF SOUND
DOVER
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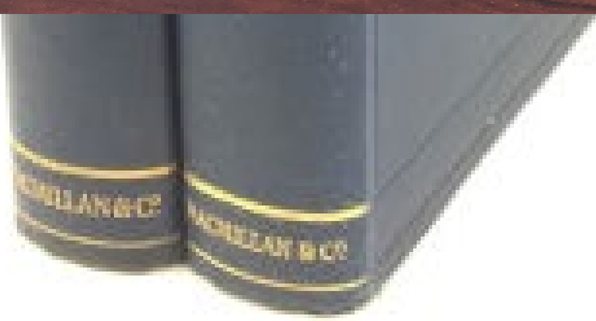
VOLUME ONE



He had a fine sense of literary style; every paper he wrote, even on the most abstruse subject, is a model of clearness and simplicity of diction. The 446 papers reprinted in his collected works clearly show **his capacity for understanding everything just a little more deeply than anyone else.**

Although a member of the House of Lords, he intervened in debate only on rare occasions, never allowing politics to interfere with science. His recreations were travel, tennis, photography and music.


SCIENTIFIC PAPERS
(John William Strutt)
LORD RAYLEIGH
Vols. I and II
*
Dover
SCIENTIFIC PAPERS
(John William Strutt)
LORD RAYLEIGH
Vols. III and IV
* *
Dover
SCIENTIFIC PAPERS
(John William Strutt)
LORD RAYLEIGH
Vols. V and VI
***
Dover

- Experiments of Rayleigh and Brace
- Rayleigh bandwidth
- Rayleigh beamwidth
- Rayleigh–Carson reciprocity
- **Rayl, rayl or Rayleigh***
- Rayleigh–Faber–Krahn inequality
- Rayleigh–Jeans law
    - Rayleigh–Jeans catastrophe
- Rayleigh–Ritz method
- Rayleigh–Schrödinger perturbation theory
- Rayleigh's method of dimensional analysis
- Rayleigh criterion
- Rayleigh's criterion (thermoacoustics)

So, what IS a Rayl?

So glad you asked.

A Rayl, rayl or Rayleigh is one of two units of specific **acoustic impedance** or, equivalently, characteristic acoustic impedance; one an MKS unit, and the other a CGS unit. These have the same dimensions as momentum per unit volume. The units are named after John William Strutt, 3rd Baron Rayleigh.

- Rayleigh–Lorentz pendulum
- Rayleigh law on low-field magnetization
- Rayleigh length
- Rayleigh limit
- Rayleigh–Gans approximation
- Rayleigh quotient
  - Rayleigh quotient iteration
  - Rayleigh's quotient in vibrations analysis

- Rayleigh test
- Rayleigh theorem
- Rayleigh theorem for eigenvalues
- Rayleigh scattering
  - Filtered Rayleigh scattering

Oops! Rayleigh limit (I forgot this earlier)

Fluid mechanics

- Rayleigh's criterion
- Janzen–Rayleigh expansion
- Plateau–Rayleigh instability
- Rayleigh–Bénard convection
- Rayleigh–Plesset equation
- Rayleigh–Taylor instability
- Rayleigh's equation (fluid dynamics)
    - Rayleigh–Kuo criterion
- Rayleigh flow
- Rayleigh number
- Rayleigh problem

Rayleigh Still

Astronomical objects

* Rayleigh 75.6°S 240.9°W, a Martian crater
- Rayleigh (lunar crater)
- 22740 Rayleigh (an asteroid)

Published: 18 March 1915

The Principle of Similitude

RAYLEIGH.

Nature **volume 95**, pages 66–68 (1915)

"I HAVE often been impressed by the scanty attention paid even by original workers in physics to the great principle of similitude. It happens not infrequently that results in the form of "laws" are put forward as novelties on the basis of elaborate experiments, which might have been predicted a *priori* after a few minutes' consideration…

However useful verification may be, whether to solve doubts or to exercise students, this seems to be an inversion of the natural order."

In engineering, applied mathematics and physics, the **Buckingham *π* theorem** is a key theorem in dimensional analysis. It is a formalization of Rayleigh's method of similitude.

Loosely, the theorem states that if there is a physically meaningful equation involving a certain number ***n* of physical variables**, then the original equation can be rewritten in terms of **a set of $p = n - k$ dimensionless parameters $\pi_1, \pi_2, ..., \pi_p$** constructed from the original variables. (Here ***k* is the number of physical dimensions*** involved; it is obtained as the rank of a particular matrix.)

* Usually [M], [L] and [T]

The Theorem

Let $q_1, q_2, q_3 \ldots q_n$ be n dimensional variables that are physically relevant in a given problem and that are inter-related by an (unknown) dimensionally homogeneous set of equations. These can be expressed via a functional relationship of the form

$$F(q_1, q_2, \ldots q_n) = 0 \qquad \text{or equivalently} \qquad q_1 = f(q_2, \ldots q_n)$$

If k is the number of fundamental dimensions required to describe the n variables, then there will be k primary variables and the remaining $j = (n-k)$ variables can be expressed as $(n-k)$ dimensionless and independent quantities or 'Pi groups', $\Pi_1, \Pi_2 \ldots \Pi_{n-k}$. The functional relationship can thus be reduced to the much more compact form:

$$\Phi(\Pi_1, \Pi_2 \ldots \Pi_{n-k}) = 0 \qquad \text{or equivalently} \qquad \Pi_1 = \phi(\Pi_2, \ldots \Pi_{n-k})$$

⇒ Note that this set of non-dimensional parameters is not unique. They are however independent and form a complete set.

Or, more briefly, for the "onto" linear transformation

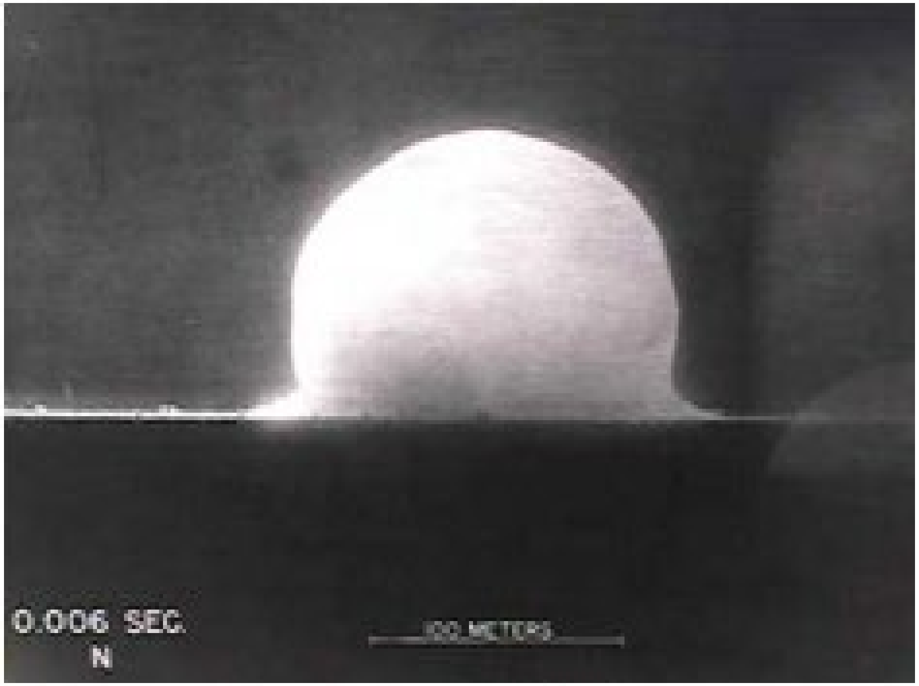
$$\mathcal{L}: \mathcal{R}^n \to \mathcal{R}^m,$$

$$rank\ (\mathcal{L}) + nullity\ (\mathcal{L}) = n$$

$$[k + (n\text{-}k) = n]$$

During World War II, the British government cooperated with the US on the development of the atomic bomb in the Manhattan project. G. I. Taylor, a British fluid dynamicist, was asked by his government to study mechanical ways of measuring the bomb’s yield (energy output).

Taylor was not directly involved in the bomb’s development, and for security reasons worked independently of the US project. He knew that the energy would be released from a small volume and would produce a very strong shock wave that would expand in approximately a spherical shape.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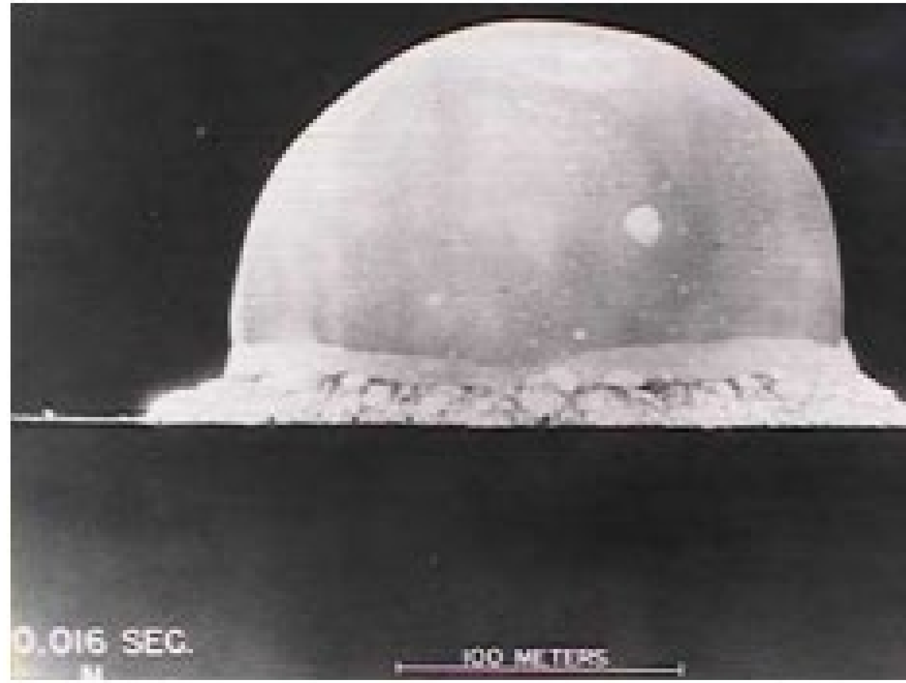
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0.053 SEC
N
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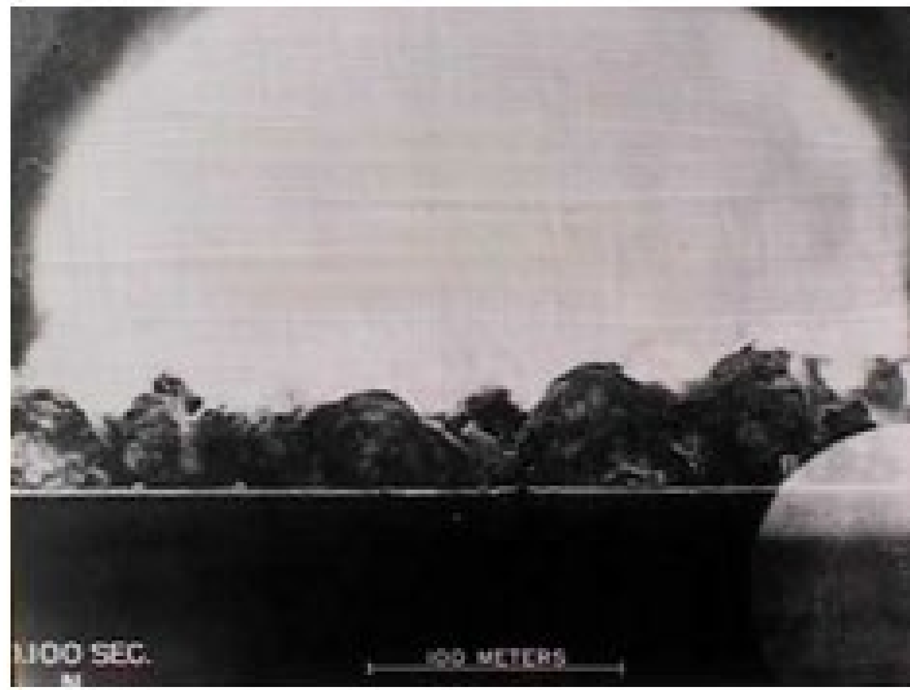

.100 SEC.
100 METERS

An example using a much-simplified version of the Theorem:

Find the radius r of the shock front (expressed as a function of time t) from an atomic explosion of energy E in an atmosphere of undisturbed density ρ. Let

$$r = K\rho^a E^b t^c,$$

(**note the implicit assumptions here**) so that in terms of the "basis vectors" [M], [L] and [T] we have

$$M^0 L^1 T^0 = (ML^{-3})^a (ML^2T^{-2})^b (T)^c.$$

Now the set $\{a, b, c\}$ is $\{-1/5, 1/5, 2/5\}$, so

$$r = K\left(\frac{Et^2}{\rho}\right)^{1/5}.$$

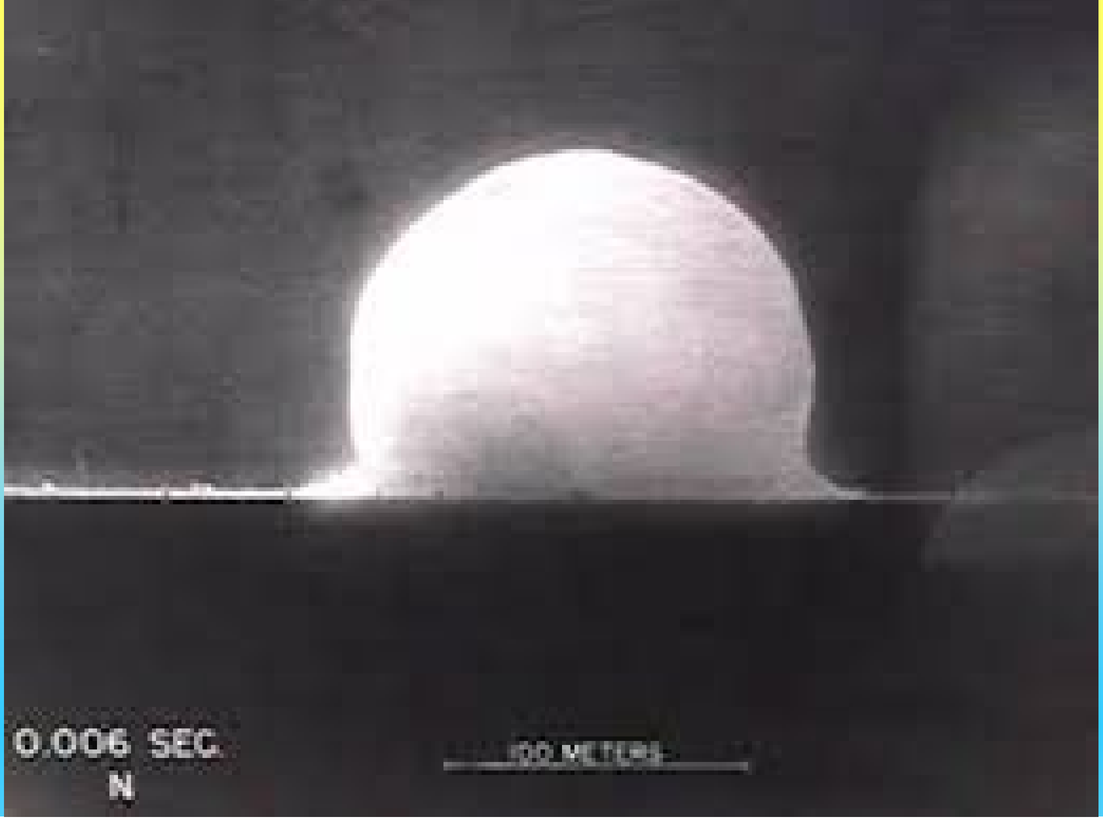
Since $r(t)$ may be determined approximately from photographic analysis of the shock front, this equation can be written in a form more useful for graphical analysis as

$$\frac{5}{2}\log r = A + \log t,$$

where the constant

$$A = \frac{5}{2}\log\left[K\left(\frac{E}{\rho}\right)^{1/5}\right].$$

If K is known (in fact it is about 1.03) then the energy of the explosion can be determined from the graph of $r(t)$ since ρ is well known (about 1.25×10^{-3} gm/cm^3 for dry air).



A more direct route: Solving the equation for E (and setting K = 1) we get:

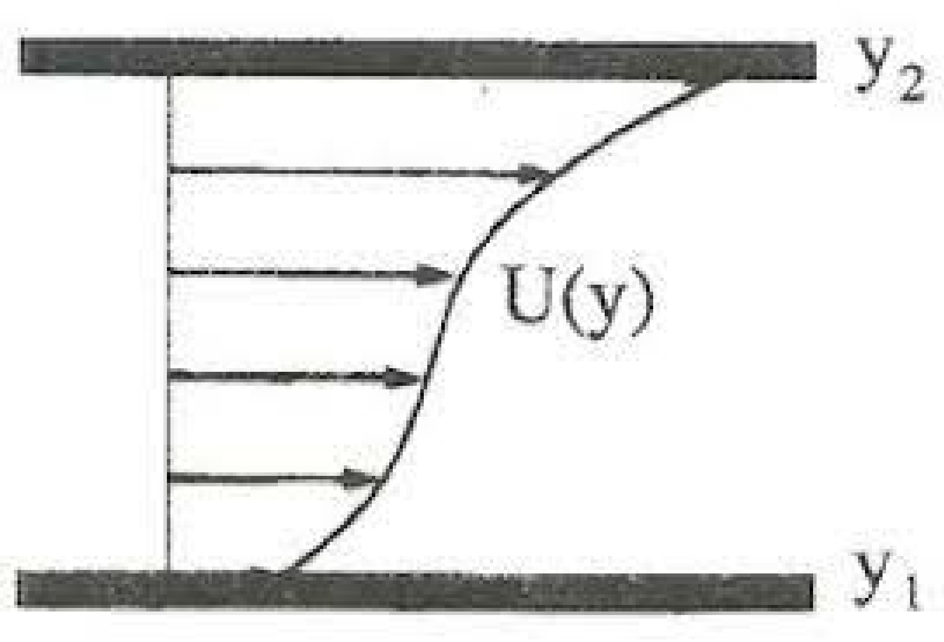
$$E = \frac{\rho r^5}{t^2}.$$

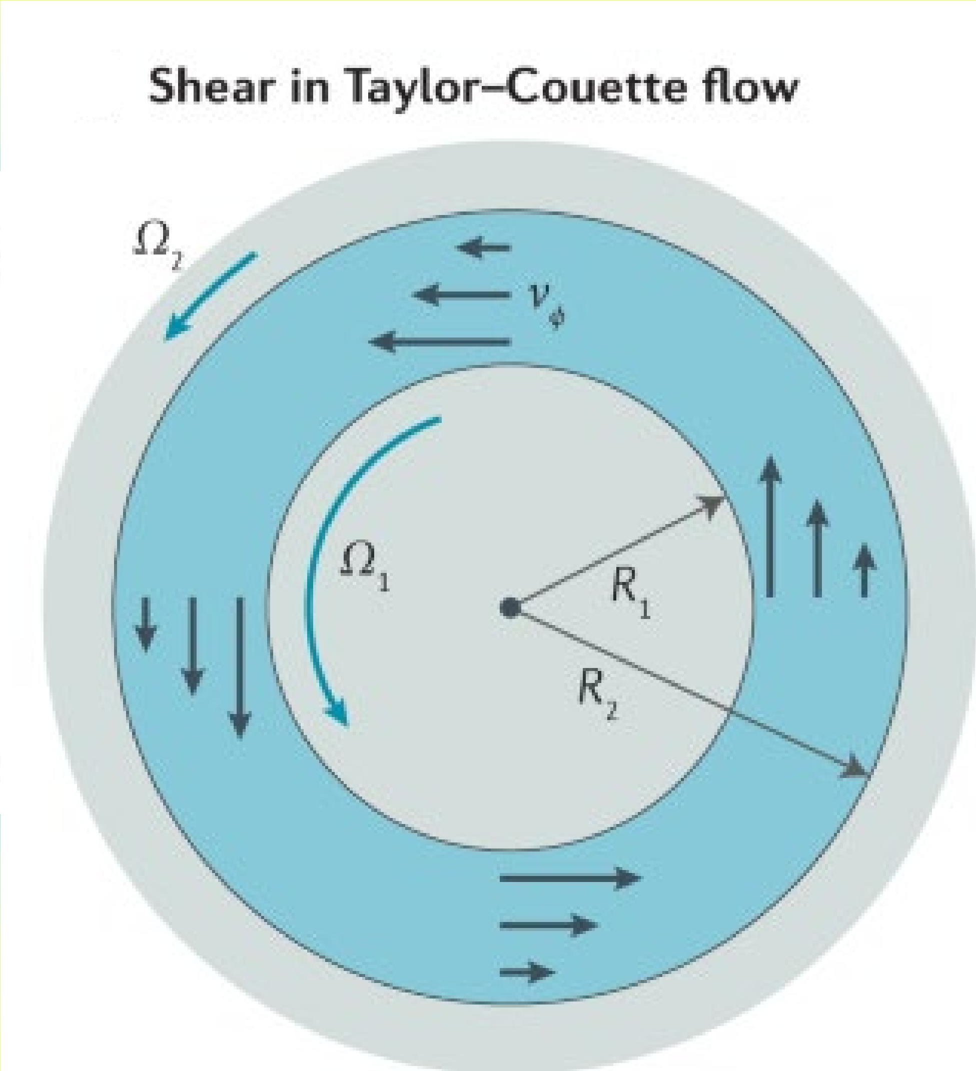
At t =.006 seconds the radius of the shock wave was approximately 80 meters. Plugging these values into the energy equation gives:

$$E \approx 10^{14} \text{ kg m}^2/\text{s}^2 = 10^{14}\text{J}.$$

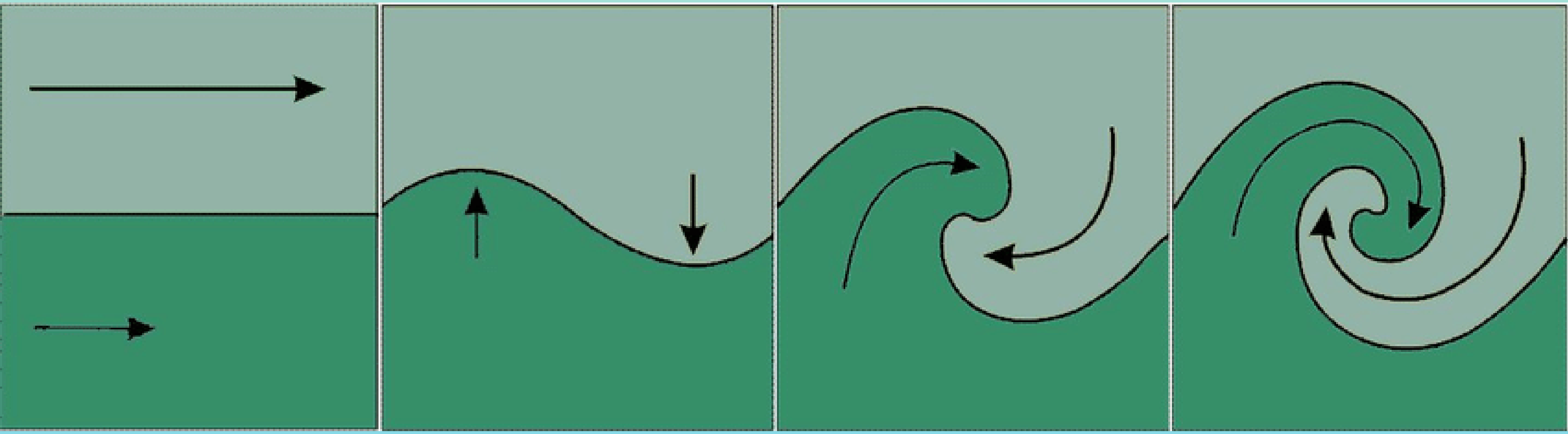
One kiloton of TNT is equivalent to $\approx 4 \times 10^{12}$J, so this was equivalent to about 25 kilotons of TNT. Apparently, this is about right!

Rayleigh's criteria for plane-parallel and radial shear flows





The **Kelvin-Helmholtz instability** – the case of a discontinuous $U(y)$




- Rayleigh Equation: Neglect viscous terms in OS equation

$$(U-c)(\mathcal{D}^2-k^2)\tilde{v}-U''\tilde{v}=0$$

with $k^2=\alpha^2+\beta^2$, and BCs: $\tilde{v}=0$ at $y=\pm 1$ at solid boundaries.

- Multiply Rayleigh equation by $\tilde{v}^*$ and integrate from $y=-1$ to $y=1$. Then integrate by parts:

$$\int_{-1}^{1} dy\,|\mathcal{D}\tilde{v}|^2+k^2|\tilde{v}|^2+\int_{-1}^{1} dy\,\frac{U''}{U-c}|\tilde{v}|^2=0$$

- Take imaginary part:

$$c_i\int_{-1}^{1} dy\,U''\frac{|\tilde{v}|^2}{|U-c|^2}=0$$

- Take imaginary part:

$$c_i \int_{-1}^{1} dy\, U'' \frac{|\tilde{v}|^2}{|U-c|^2} = 0$$

- Both $|\tilde{v}|^2$ and $|U-c|^2$ are nonnegative. If c_i is positive, *then* U'' has to change sign in order for the integral to be zero.

Rayleigh's inflexion point theorem

If there exist perturbations with $c_i > 0$, then $U''(y)$ must vanish for some $y \in [-1, 1]$ for instability.

Thus, the vorticity $U'(y)$ must have an extremum within the flow. This is a necessary condition for linear instability.

Rayleigh Scattering

filtrate evaporated to dryness, gently ignited, and the mixed chlorides weighed.

The separation of potassa and soda was effected in the usual way by means of chloride of platinum.

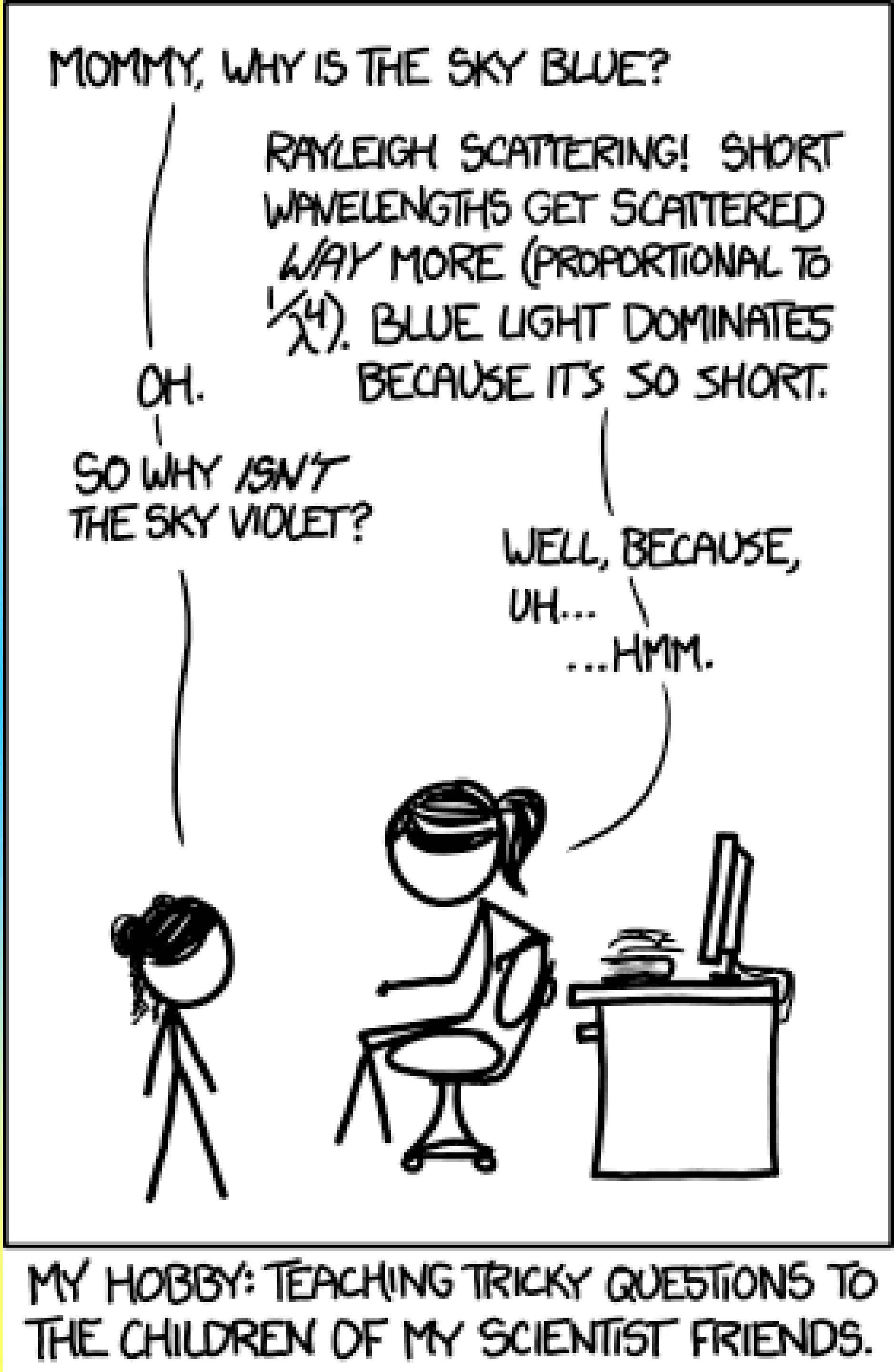
XV. *On the Light from the Sky, its Polarization and Colour.*

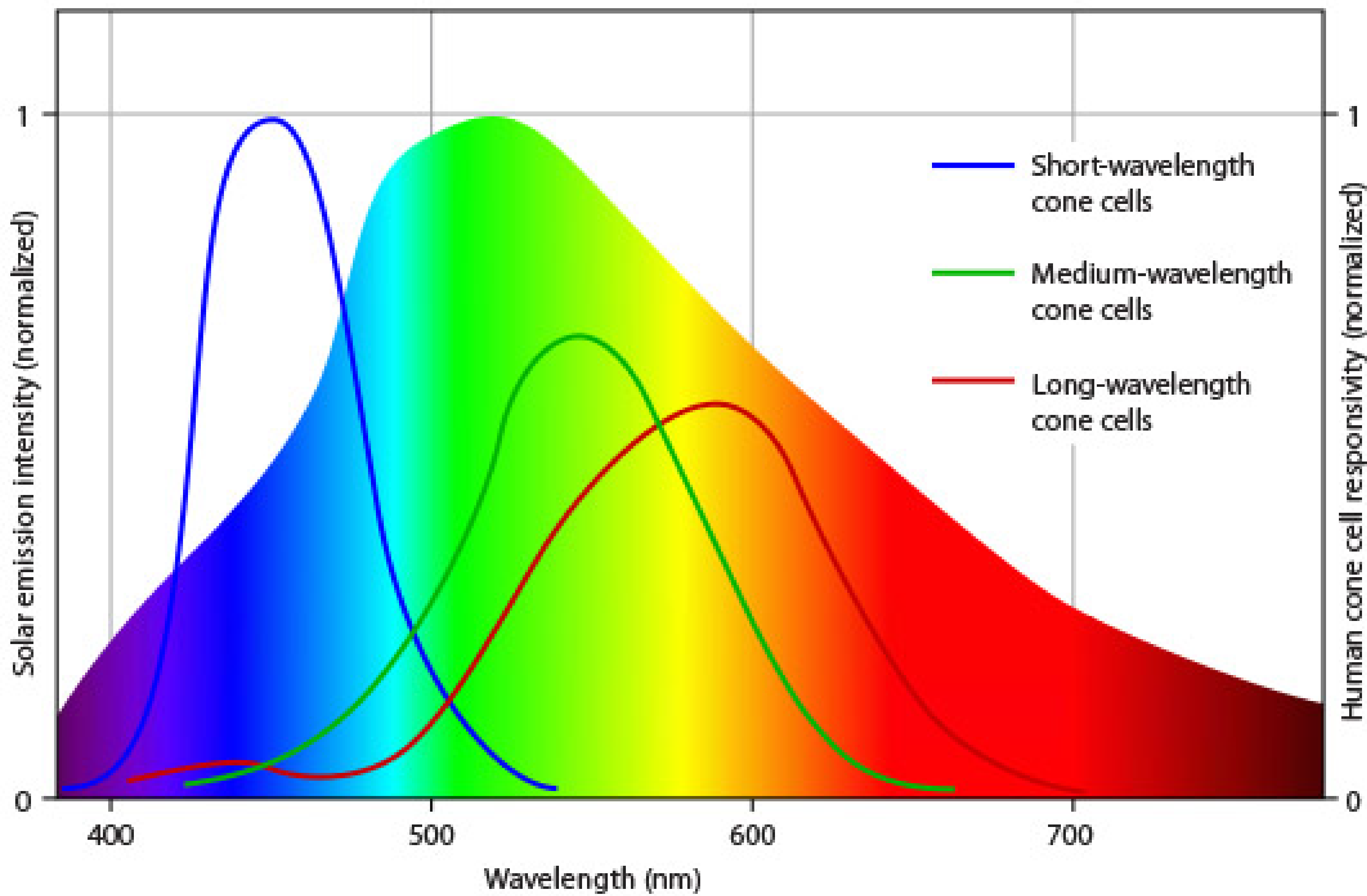
By the Hon. J. W. Strutt, *Fellow of Trinity College, Cambridge**.

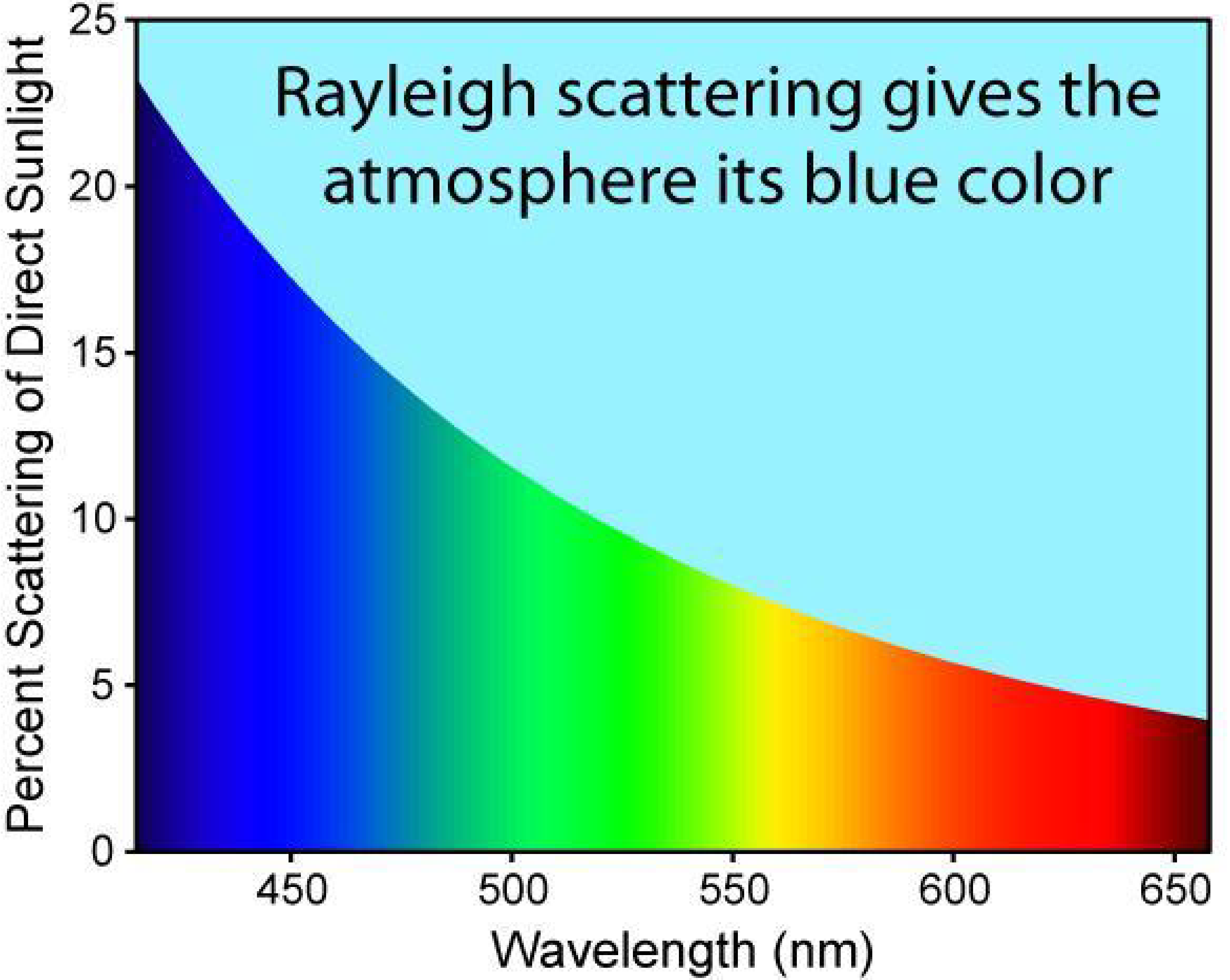
It is now, I believe, generally admitted that the light which we receive from the clear sky is due in one way or another to small suspended particles which divert the light from its regular course. On this point the experiments of Tyndall with precipitated clouds seem quite decisive. Whenever the particles of the foreign matter are sufficiently fine, the light emitted laterally is blue in colour, and, in a direction perpendicular to that of the incident beam, is *completely polarized.*

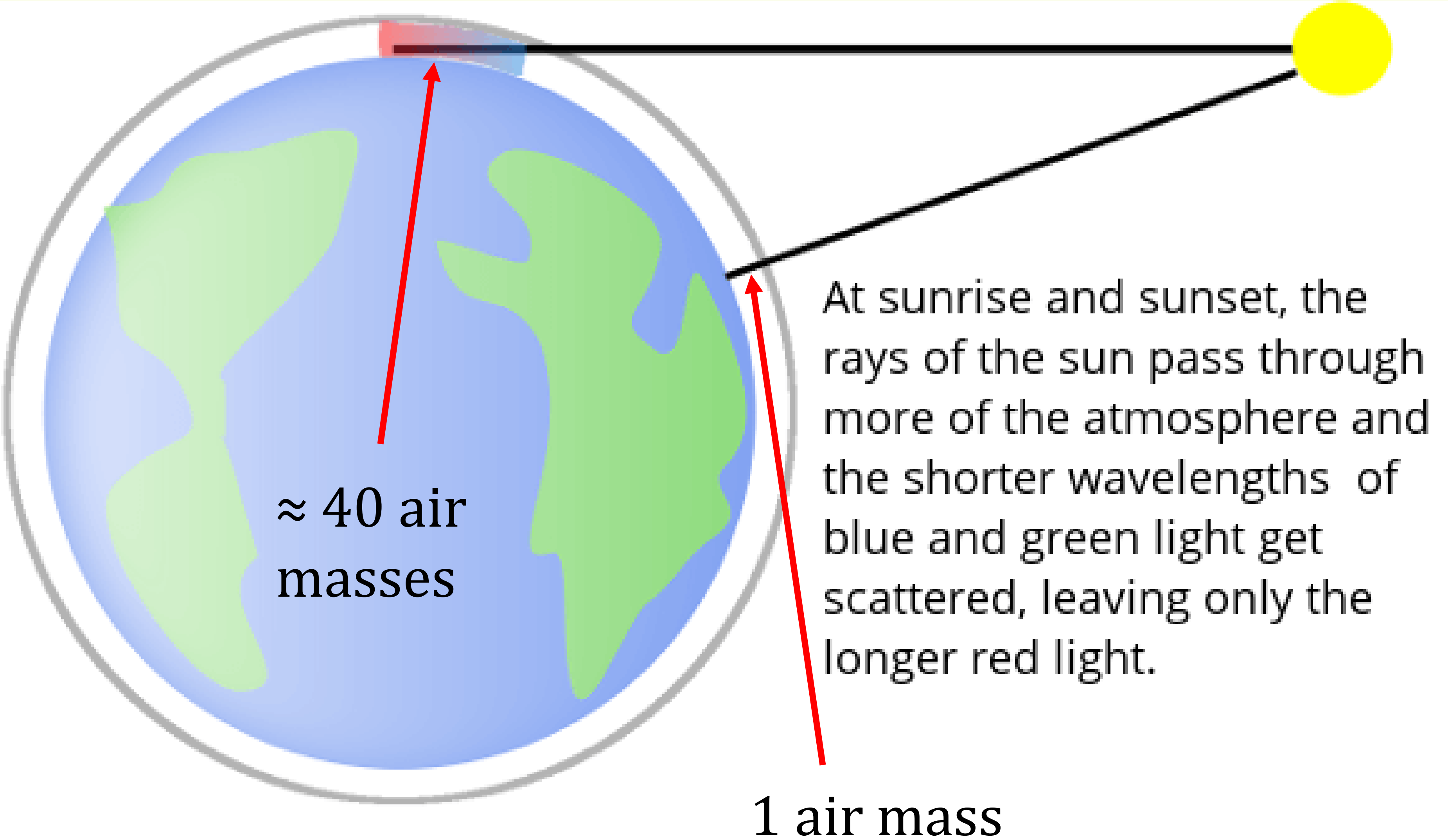
About the colour there is no *prima facie* difficulty; for as soon as the question is raised, it is seen that the standard of linear dimension, with reference to which the particles are called small, is the wave-length of light, and that a given set of particles would (on any conceivable view as to their mode of action) produce a continually increasing disturbance as we pass along the spectrum towards the more refrangible end; and there seems no reason why the colour of the compound light thus scattered laterally should not agree with that of the sky.

Simplistic version




Short-wavelength cone cells
Medium-wavelength cone cells
Long-wavelength cone cells
Solar emission intensity (normalized)
Human cone cell responsivity (normalized)
Wavelength (nm)
1
0
400
500
600
700
1
0


Rayleigh scattering gives the
atmosphere its blue color
Percent Scattering of Direct Sunlight
0
5
10
15
20
25
450
500
550
600
650
Wavelength (nm)

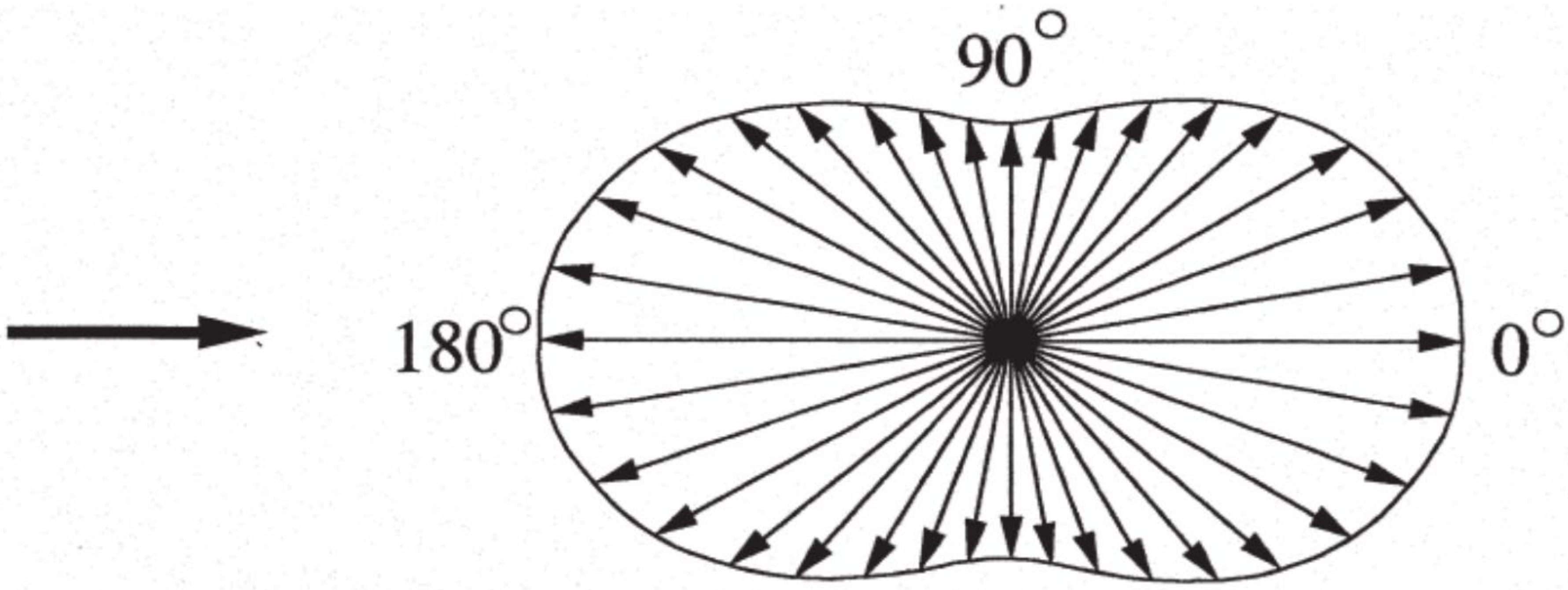


At sunrise and sunset, the rays of the sun pass through more of the atmosphere and the shorter wavelengths of blue and green light get scattered, leaving only the longer red light.

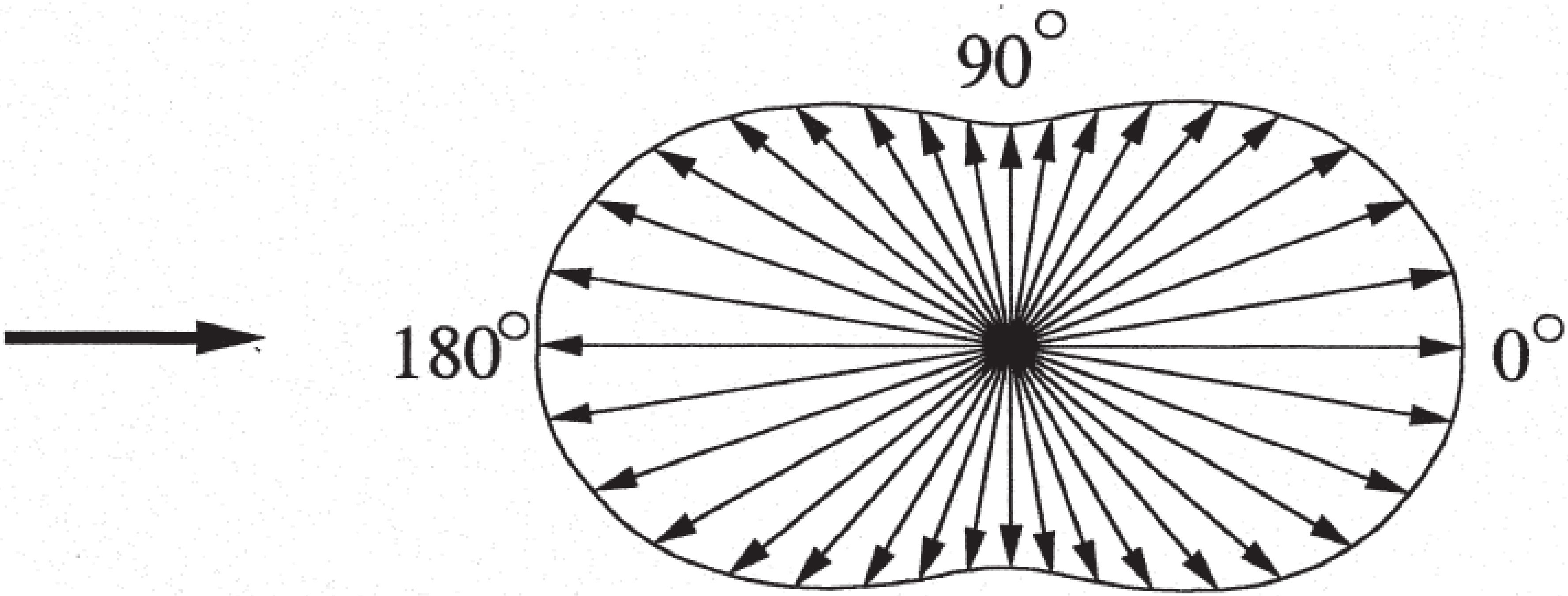
So why IS the sky **blue**?

$$I = I_0 \frac{1+\cos^2\theta}{2R^2}\left(\frac{2\pi}{\lambda}\right)^4\left(\frac{n^2-1}{n^2+2}\right)^2\left(\frac{d}{2}\right)^6$$

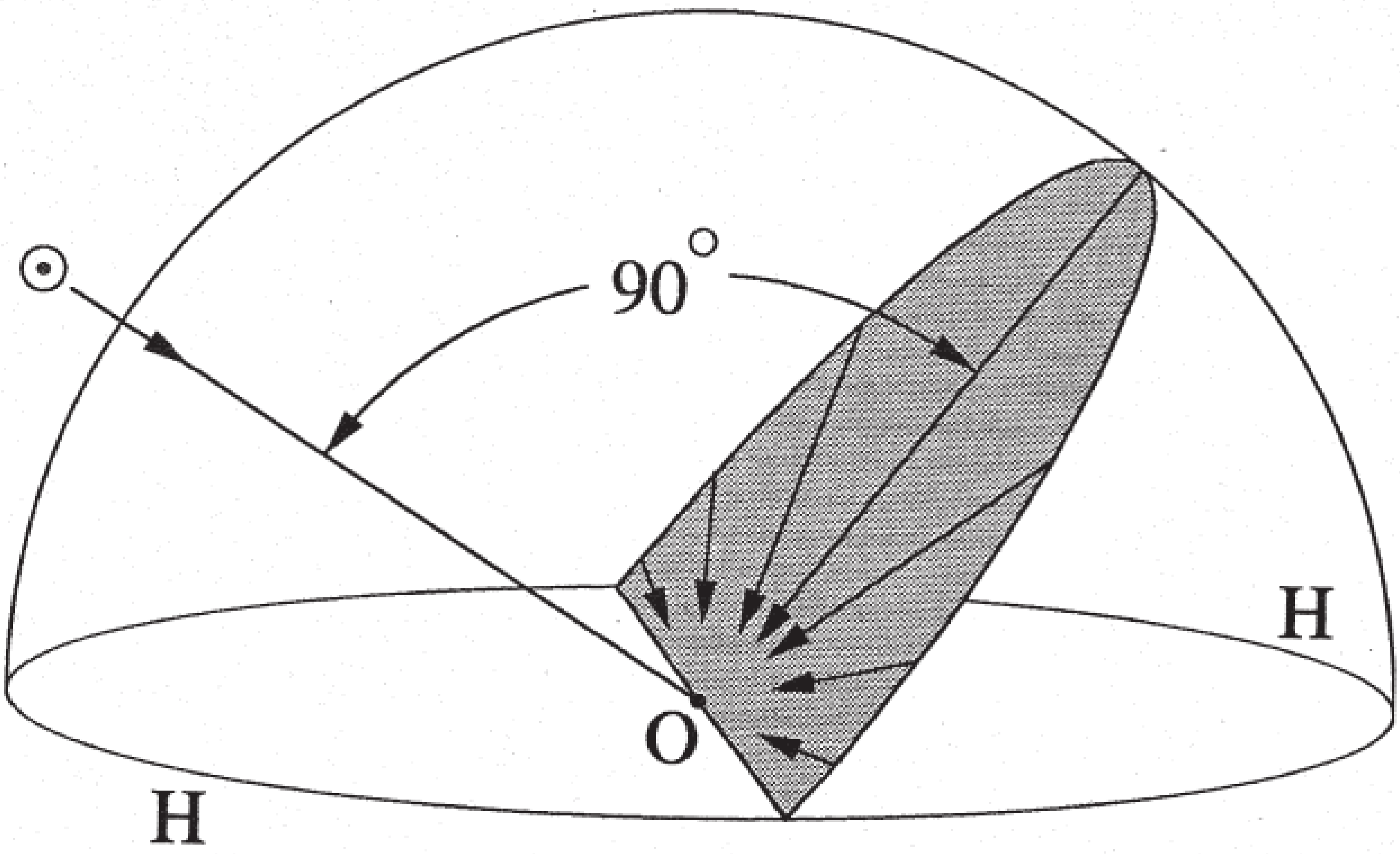
Of course!



Tiny particles also scatter light more or less equally in all directions (though somewhat less light is scattered by 90°, where it is polarized).



The sky is most strongly polarized 90° from the Sun.



On Rayleigh and Mie scattering

Jerald W. Caruthers

Citation: Proc. Mtgs. Acoust. **14**, 070001 (2011); doi: 10.1121/1.3664646
View online: https://doi.org/10.1121/1.3664646
View Table of Contents: https://asa.scitation.org/toc/pma/14/1
Published by the Acoustical Society of America

What is "Rayleigh Scattering"?

"Rayleigh Scattering" has come to mean that scattering theory describing a form that is far short of what Rayleigh actually contributed to scattering theory.

That form is $(ka)^4$ for $ka \ll 1$, or $a \ll \lambda / 2\pi$.

For the optical scientist, "Rayleigh Scattering" explains the blue sky.

Relegating "Rayleigh scattering" to this simpler phenomenon is unfortunate!

What is Mie Scattering?

"Mie scattering" is a complete and precise electromagnetic theory for scattering of a plane wave from a sphere of any size.

It involves the solution of the scalar wave equation in spherical coordinates:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}+k^2r^2\psi=0 \qquad (1)$$

where $\psi(r,\theta,\phi)$, as we will show, is a *scalar potential of either the electric- or the magnetic-field vector.*

Solutions: Legendre functions, spherical harmonics, and *modified Bessel functions of half integer order* (now symbolicly different and generally called *spherical Bessel functions*).[3]

Mie (1908) was not the first or only to develop this theory: Ludwig Lorenz in 1890[4] and Peter Debye in 1909[5], and others.

Rayleigh's 1872/1878 Scattering Theory

Rayleigh published a complete and precise scattering theory beginning in 1872*[6] and in 1878 in the 1st edition of his book, *The Theory of Sound.*[7]

This Rayleigh theory was presented in the sections from §323 through §334.

These sections have been ignored![8]

Rayleigh was the first to provide a complete and precise solution to the scalar wave equation in spherical coordinate. *I repeat the equation here for emphasis*:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}+k^2r^2\psi=0 \qquad (1)$$

where $\psi(r,\theta,\phi)$ is the *scalar potential of the particle-velocity vector*.

Rayleigh's solution, like Mie's, resulted in a closed form involving Legendre functions, spherical harmonics, and *spherical Bessel functions* (given in their modern form).

Also, Rayleigh actually *advanced the mathematics* developing in the era with his solution to this problem.

§ 323, if ψ be independent of r, as it is evident that it must approximately be in the case supposed, we have

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\psi}{d\theta}\right)+\frac{1}{\sin^2\theta}\frac{d^2\psi}{d\omega^2}+k^2r^2\psi=0 \;\ldots\ldots (3),$$

whose solution is simply

$$\psi_n = S_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4),$$

while the admissible values of k^2 are given by

$$k^2r^2 = n(n+1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5).$$

The interval between the gravest tone ($n = 1$) and the next is such that two of them would make a twelfth (octave + fifth). The problem of the spherical sheet of gas will be further considered in the following chapter. [For a derivation of (5) from the fundamental determinant, equivalent to (1), the reader may be referred to a short paper[1] by Mr Chree.]

334. The next application that we shall make of the spherical harmonic analysis is to investigate the disturbance which ensues when plane waves of sound impinge on an obstructing sphere. Taking the centre of the sphere as origin of polar co-ordinates, and the direction from which the waves come as the axis of μ, let ϕ be the potential of the unobstructed plane waves. Then, leaving out an unnecessary complex coefficient, we have

$$\phi = e^{ik(at+x)} = e^{ikat}\,.\,e^{ikr\mu} \ldots\ldots\ldots\ldots\ldots\ldots (1),$$

and the solution of the problem requires the expansion of $e^{ikr\mu}$ in spherical harmonics. On account of the symmetry the harmonics reduce themselves to Legendre's functions $P_n(\mu)$, so that we may take

$$e^{ikr\mu} = A_0 + A_1P_1 + \ldots + A_nP_n + \ldots\ldots\ldots\ldots (2),$$

where $A_0 \ldots$ are functions of r, but not of μ. From what has been already proved we may anticipate that A_n, considered as a function of r, must vary as

$$P_n\left(\frac{d}{d\,.\,ikr}\right)\frac{\sin kr}{kr}, \quad \text{or as} \quad r^{-\frac12}J_{n+\frac12}(kr),$$

but the same result may be easily obtained directly. Multiplying

[1] *Messenger of Mathematics*, vol. xv. p. 20, 1886.

(2) by $P_n(\mu)$, and integrating with respect to μ from $\mu = -1$ to $\mu = +1$, we find

$$\int_{-1}^{+1}P_n(\mu)\,e^{ikr\mu}\,d\mu = A_n\int_{-1}^{+1}(P_n)^2\,d\mu = \frac{2A_n}{2n+1} \ldots\ldots\ldots (3);$$

and, as in § 330,

$$\int_{-1}^{+1}P_n(\mu)\,e^{ikr\mu}\,d\mu = 2P_n\left(\frac{d}{d\,.\,ikr}\right)\,.\,\frac{\sin kr}{kr},$$

so that finally

$$\frac{A_n}{2n+1} = P_n\left(\frac{d}{d\,.\,ikr}\right)\frac{\sin kr}{kr} = i^n\sqrt{\frac{\pi}{2kr}}\,.\,J_{n+\frac12}(kr) \ldots\ldots (4).$$

In the problem in hand the whole motion outside the sphere may be divided into two parts; the first, that represented by ϕ and corresponding to undisturbed plane waves, and the second a disturbance due to the presence of the sphere, and radiating outwards from it. If the potential of the latter part be ψ, we have (2) § 324 on replacing the general harmonic S_n by $a_nP_n(\mu)$,

$$\left.\begin{aligned} r\psi_n &= a_n P_n(\mu)\,.\,e^{-ikr}f_n(ikr) \\ r^2\frac{d\psi_n}{dr} &= -a_n P_n(\mu)\,.\,e^{-ikr}F_n(ikr)\end{aligned}\right\} \ldots\ldots\ldots\ldots (5).$$

The velocity-potential of the whole motion is found by addition of ϕ and ψ, the constants a_n being determined by the boundary conditions, whose form depends upon the character of the obstruction presented by the sphere. The simplest case is that of a rigid and fixed sphere, and then the condition to be satisfied when $r = c$ is that

$$\frac{d\phi}{dr}+\frac{d\psi}{dr} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6),$$

a relation which must of course hold good for each harmonic element separately. For the element of order n, we get

$$a_n = (2n+1)\frac{kc^2e^{ikc}}{F_n(ikc)}P_n\left(\frac{d}{d\,.\,ikc}\right)\frac{d}{d\,.\,kc}\,.\,\frac{\sin kc}{kc} \ldots\ldots\ldots (7).$$

Corresponding to the plane waves $\phi = e^{ik(at+x)}$, the disturbance due to the presence of the sphere is expressed by

$$\psi = \frac{kc^2}{r}e^{ik(at-r+c)}$$

$$\times\sum_{n=0}^{n=\infty}\frac{2n+1}{F_n(ikc)}P_n\left(\frac{d}{d\,.\,ikc}\right)\frac{d}{d\,.\,kc}\,.\,\frac{\sin kc}{kc}\,.\,P_n(\mu)\,.\,f_n(ikr) \ldots (8).$$

Some of Rayleigh's Contributions to the Mathematics of the Time

In his treatment of plane-wave scattering from a sphere in 1878, Rayleigh:*

- was the first to expand plane waves in spherical coordinates.[9]
- was the first to introduce spherical Bessel functions to this problem.
- used Stokes functions as series solutions and showed their relationship to spherical Bessel functions.[10,11,12]
- created a generating function for spherical Bessel functions.[13,14,15]
- showed that Bessel functions are special cases of Laplace functions.[16]

Sources

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John William Strutt, 3rd Baron Rayleigh – Wikipedia.
John William Strutt (1842 - 1919) - Biography - MacTutor History of Mathematics.
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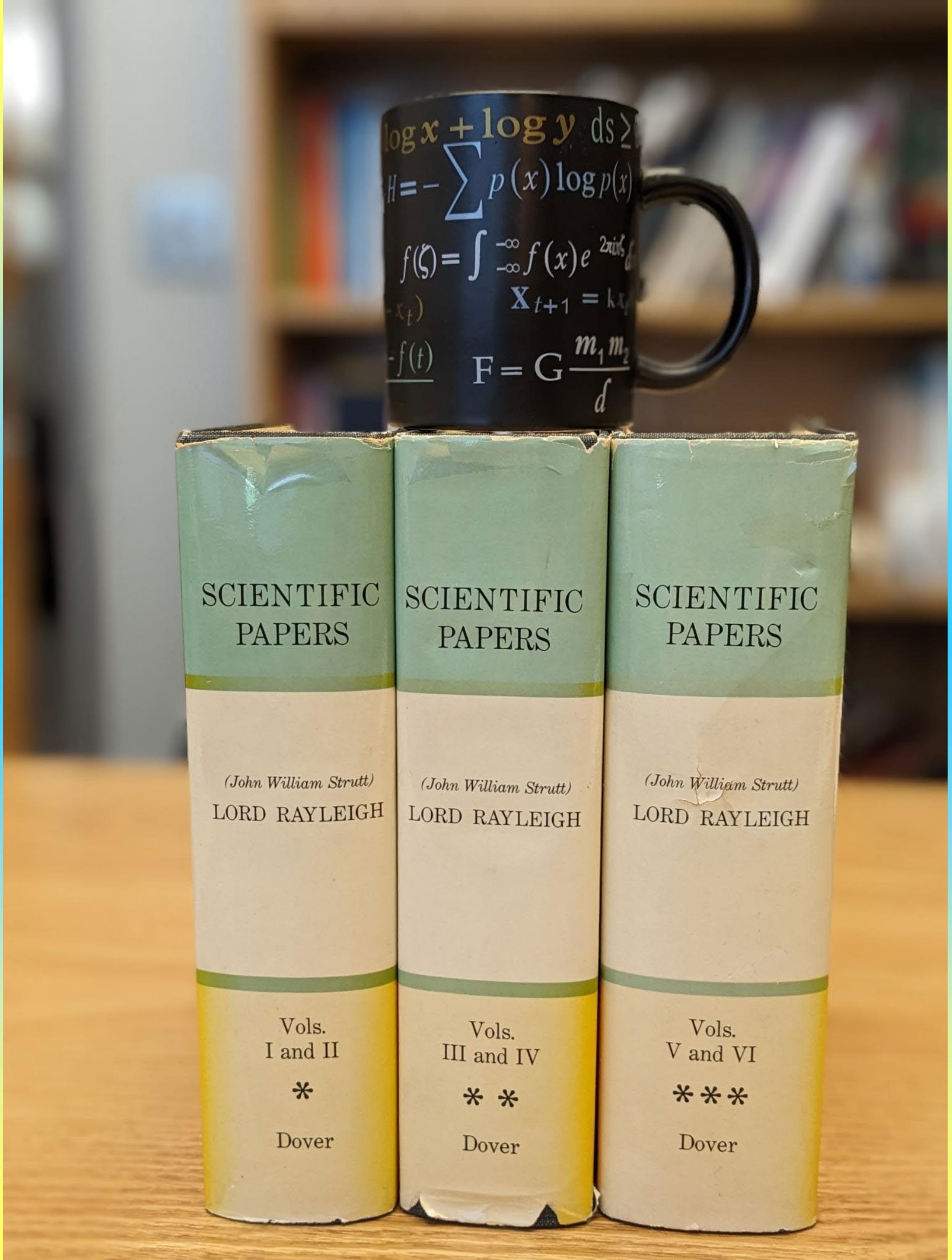

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Dover
SCIENTIFIC PAPERS
(John William Strutt)
LORD RAYLEIGH
Vols. III and IV
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Dover
SCIENTIFIC PAPERS
(John William Strutt)
LORD RAYLEIGH
Vols. V and VI
* * *
Dover

Question 1: (i) Estimate the total number of (numbered) pages in all six volumes.

(ii) Estimate the total numbers of words in all six volumes.

Question 2: Estimate the combined mass of all three tomes.

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fire escape
keep clear

66.

ON THE STABILITY, OR INSTABILITY, OF CERTAIN FLUID MOTIONS.

[*Proceedings of the London Mathematical Society*, XI. pp. 57—70, 1880.]

In a former communication to the Society on the "Instability of Jets*," I applied a method due to Sir W. Thomson, to calculate the manner of falling away from equilibrium of jets bounded by one or more surfaces of discontinuity. Such interest as these investigations possessed was due principally to the possibility of applying their results to the explanation of certain acoustical phenomena relating to sensitive flames and smoke jets. But it soon appeared that in one important respect the calculations failed to correspond with the facts.

To fix the ideas, let us take the case of an originally plane surface of separation, on the two sides of which the fluid moves with equal and opposite constant velocities ($\pm V$). In equilibrium the elevation h, at every point x along the surface, is zero. It is proved that, if initially the surface be at rest in the form defined by $h = H \cos kx$, then, after a time t, its form is given by

$$h = H \cos kx \ \cosh kVt, \quad \ldots\ldots(1)$$

provided that, throughout the whole time contemplated, the disturbance is small. In the same sense as that in which the frequency of vibration measures the stability of a system vibrating about a configuration of stable equilibrium, so the coefficient kV of t, in equations such as (1), measures the instability of an unstable system; and we see, in the present case, that the instability increases without limit with k; that is to say, the shorter the wave-length of the sinuosities on the surface of separation, the more rapidly are they magnified.

The application of this result to sensitive jets would lead us to the conclusion that their sensitiveness increases indefinitely with pitch. It is

* *Proceedings*, vol. x. p. 4, Nov. 14, 1878. [Art. LVIII.]

true that, in the case of certain flames, the pitch of the most efficient sounds is very high, not far from the upper limit of human hearing; but there are other kinds of sensitive jets on which these high sounds are without effect, and which require for their excitation a moderate or even a grave pitch.

A probable explanation of the discrepancy readily suggests itself. The calculations are founded upon the supposition that the changes of velocity are discontinuous—a supposition which cannot possibly agree with reality. In consequence of fluid friction a surface of discontinuity, even if it could ever be formed, would instantaneously disappear, the transition from the one velocity to the other becoming more and more gradual, until the layer of transition attained a sensible width. When this width is comparable with the wave-length of the sinuosity, the solution for an abrupt transition ceases to be applicable, and we have no reason for supposing that the instability would increase for much shorter wave-lengths.

In the following investigations, I shall suppose that the motion is entirely in two dimensions, parallel (say) to the plane xy, so that (in the usual notation) w is zero, as well as the rotations ξ, η. The rotation ζ parallel to z is connected with the velocities u, v by the equation

$$\zeta = \tfrac{1}{2}\left(\frac{du}{dy} - \frac{dv}{dx}\right). \quad \ldots\ldots(2)$$

When the phenomena under consideration are such that the compressibility may be neglected, the condition that no fluid is anywhere introduced or abstracted, gives

$$\frac{du}{dx} + \frac{dv}{dy} = 0. \quad \ldots\ldots(3)$$

In the absence of friction, ζ remains constant for every particle of the fluid; otherwise, if ν be the kinematic viscosity, the general equation for ζ is

$$\frac{\partial\zeta}{\partial t} = \xi\frac{dw}{dx} + \eta\frac{dw}{dy} + \zeta\frac{dw}{dz} + \nu\nabla^2\zeta^*, \quad \ldots\ldots(4)$$

where

$$\partial/\partial t = d/dt + u\,d/dx + v\,d/dy + w\,d/dz, \quad \ldots\ldots(5)$$

and

$$\nabla^2 = d^2/dx^2 + d^2/dy^2 + d^2/dz^2. \quad \ldots\ldots(6)$$

For the proposed applications to motion in two dimensions, these equations reduce to

$$\partial\zeta/\partial t = \nu\nabla^2\zeta, \quad \ldots\ldots(7)$$

$$\partial/\partial t = d/dt + u\,d/dx + v\,d/dy, \quad \ldots\ldots(8)$$

$$\nabla^2 = d^2/dx^2 + d^2/dy^2, \quad \ldots\ldots(9)$$

while the two other equations similar to (4) are satisfied identically.

* Lamb's *Motion of Fluids*, p. 243.

Question 1: (i) Estimate the total number of (numbered) pages in all six volumes.

The coffee mug is of a typical size: diameter ≈ 8cm, height ≈ 10 cm. the thickness of each book varies slightly, but we can ignore that. Therefore, each tome, by comparison, is about 6.5 cm thick and 25 cm high. The width can be estimated (for books about this size) as ~ 15 cm.

Typically, on my bookshelf there are ~ 200 numbered pages/cm (i.e., ~ 100 individual pages), so the number of pages in all six volumes (three tomes) ~ $3 \times 6.5 \times 200 \approx 3900$ pages.

(ii) Estimate the total numbers of words in all six volumes. Assume for simplicity that each page is covered uniformly with words (i.e., consider the space occupied by equations to be replaced by words).

(ii) A typical page (replacing equations by writing) has ~ 40 lines with ~ 12 words/line, or ~ 500 words/page. Hence the number of words in total is ~ $3900\times500 \approx 2\times10^6$.

Question 2: Estimate the combined mass of all three tomes.

The density of paper varies, but it does float on water initially…until it absorbs water. So, its density is less than that of water…

For typical academic books (the Dover edition was published in 1964) it is ~ 800 kg/m^3. The volume of each of the three books is ~ $6.5 \times 15 \times 25$ cm^3 ≈ 2400 cm^3 = 2.4×10^{-3} m^3. Hence the total volume is ~ 7×10^{-3} m^3. Hence the total mass is ~ 8×10^2 kg/m^3 × 7×10^{-3} m^3 = 5.6 kg.

In fact, there are approximately 3700 pages and the whole set has a mass of 5.4 kg (using my wife's kitchen scales).

I didn't count the number of words because I'm working on my next Fermi column!

Question: Will Lord Rayleigh's weighty words sink if I drop them in the ODU pond?