Measuring Identification Risk in Microdata Release and Its Control by Post-randomization

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What is Statistical Disclosure Control

Disclosure Control

research the issues of privacy and confidentiality that arise in the process collecting data from the public and disclosing the data to a certain group of people.

Statistical Disclosure Control

explores disclosure control issues from a statistical point of view, including (but not limited to) proper measures of privacy and confidentiality, statistical techniques of perturbing the microdata, inference after the perturbation, etc.
Why SDC

- The law requires confidentiality to be preserved, even without publishing the data.
- The need for sharing more microdata with public is becoming stronger than ever.
- Inference issues with the perturbed data.
Previous Work


Our Contribution

- We focus on identity disclosure based on categorical key variables.
- A new measure for identification risk and a associated disclosure control goal.
- A method that accomplishes the preceding goal, using Post-Randomization (PRAM).
- Effects of our method upon the inference issues.
Definition of Identity Disclosure

Assumptions

- Intruder knows the key variable value of the target.
- Units are non-differentiable with the same key variable values, and the intruder would pick one at random as the record of the target.

Identity Disclosure: Correct Match

A correct match happens to a unit when the intruder correctly matches the unit’s record of non-key variable value, among all the units that share the same key variable value with the target.

We measure the risk of identity disclosure by the probability of a unit being correctly matched.
For example,
If the original data is released after the removal of names,

\[ P\{\text{John is correctly matched}\} = 1 \]
\[ P\{\text{Susan is correctly matched}\} = 0.5 \]
Disclosure Control Goal

$$P(CM|S_j = a, X_B = c_j) \leq \xi$$

for all $a > 0$ and $j = 1, 2, \ldots, k$.

- $CM$ stands for the event that the target unit $B$ is correctly matched in the aforementioned scenario and matching scheme.
- $c_1, c_2, \ldots, c_k$ are all the cells (values of the cross-classified variable formed by all key variables).
- $S_j$ is the count of $c_j$ in the perturbed released data.
- The intruder knows the target’s key variable value, $X_B = c_j$. 
Our Approach: Post-Randomization (PRAM)

What is PRAM

In a nutshell, PRAM is the randomization mechanism of a categorical variable using a transition probability where the transition probability is a function of the data, instead of being predetermined.

EX: A Bernoulli dataset with 10 observations $X_1, X_2, \ldots, X_{10}$. A PRAM transition matrix could be

$$ P = \begin{pmatrix} 1 - \frac{1}{T_0} & \frac{1}{T_0} \\ \frac{1}{T_1} & 1 - \frac{1}{T_1} \end{pmatrix} $$

where $T_i$ is the count of $i$, and $p_{ij} = P\{j \rightarrow i\}$. If $X_1 = 0$, then change $X_1$ to 1 with probability $\frac{1}{T_0}$. 
Our Approach: Post-Randomization (PRAM)

Our choice of PRAM matrix  Let a group contain cells \( c_1, c_2, \ldots, c_k \). Then the transition probability matrix is \( P = ((p_{ij})) \) where

\[
p_{ii} = 1 - \frac{\theta}{T_i}, \quad p_{ji} = \frac{\theta}{(k - 1)T_i}
\]

for \( i, j = 1, 2, \ldots, k \), \( i \neq j \), \( 0 \leq \theta \leq 1 \), and \( T_i \) is the count of \( c_i \) in the original dataset.
Our Approach: Post-Randomization (PRAM)

Physical interpretation of $\theta$:

\[
E(\text{number of units moving out of cell } i) = T_i - E(\text{number of units of does not change in cell } i) = T_i - T_i \times p_{ii} = \theta
\]

Being independent of $\theta$, this applies to all cells.
Our Approach: Post-Randomization (PRAM)

Example:

<table>
<thead>
<tr>
<th>Name</th>
<th>Sex</th>
<th>Race</th>
<th>Residency</th>
<th>VIN</th>
<th>Cross-classification of keys</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>M</td>
<td>White</td>
<td>VA</td>
<td>a</td>
<td>c1</td>
</tr>
<tr>
<td>Mike</td>
<td>M</td>
<td>Black</td>
<td>MD</td>
<td>b</td>
<td>c2</td>
</tr>
<tr>
<td>Larry</td>
<td>M</td>
<td>Black</td>
<td>VA</td>
<td>c</td>
<td>c3</td>
</tr>
<tr>
<td>Susan</td>
<td>F</td>
<td>White</td>
<td>MD</td>
<td>d</td>
<td>c4</td>
</tr>
<tr>
<td>Jane</td>
<td>F</td>
<td>Other</td>
<td>MD</td>
<td>e</td>
<td>c5</td>
</tr>
<tr>
<td>Rachel</td>
<td>F</td>
<td>White</td>
<td>MD</td>
<td>f</td>
<td>c4</td>
</tr>
</tbody>
</table>

\[
\begin{pmatrix}
  c_1 & c_2 & c_3 & c_4 & c_5 \\
  1 - \theta & \theta/4 & \theta/4 & \theta/8 & \theta/4 \\
  \theta/4 & 1 - \theta & \theta/4 & \theta/8 & \theta/4 \\
  \theta/4 & \theta/4 & 1 - \theta & \theta/8 & \theta/4 \\
  \theta/4 & \theta/4 & \theta/4 & 1 - \theta & \theta/4 \\
  \theta/4 & \theta/4 & \theta/4 & \theta/8 & 1 - \theta \\
\end{pmatrix}
\]
Pros and cons of our approach

Pro:

- Easy to operate: reducing the choosing matrix problem to choosing one parameter for each group
- Unbiased estimators: $E(S_i|T_i) = T_i$
- PRAM matrix, being dependent on the original data, is hard to retrieve;

Con:

- Simple structure costs unnecessary perturbation
- limited to $\xi \geq \frac{1}{3}$
Our perturbation mechanism

- We set $\xi \geq \frac{1}{3}$
- Solve for a common $\theta$ for all group.
- Solve for the minimum group size $k$.
- Subset only the singleton and doubleton cells. Partition the subset into groups of at least $k$ cells.
- PRAM each group independently.
Solution of $\theta$ and $k$

ultimate goal: $P(CM|S_j = a, X_B = c_j) \leq \xi$

$\uparrow$

$P(CM|S_j = a, X_B = c_j, T = t) \leq \xi$, where $T$ is the vector of all cells’ counts

$\uparrow$

$P(CM|S_j = 1, X_B = c_j, T = t) \leq \xi$, $P(CM|S_j = 2, X_B = c_j, T = t) \leq \xi$,

$\xi \geq \frac{1}{3}$

$\uparrow$

$P(CM|S_j = 2, X_B = c_j, T = t) \leq P(CM|S_j = 1, X_B = c_j, T = t) \leq \phi(\theta)$

$\phi(\theta) = \phi(\theta) = \frac{T_j - \theta}{T_j(T_j - \theta) + \theta^2} \leq \xi$

Solution

Solve $\phi(\theta) \leq \xi$ for $\theta$. Then plug $\theta$ in $P(CM|S_j = 2, X_B = c_j, T = t) \leq P(CM|S_j = 1, X_B = c_j, T = t)$ to solve the smallest possible $k$. 
The exploration on data quality serves mostly as a guide of how to partition all categories into groups, so that the groups are formed in the way that it has a total variation as small as possible.

**Numerical findings:**

- Total variation from perturbing using PRAM, i.e.
  \[ \sum \text{var}(S_i|T_i), \]
  is ignorable compared to the total variation from sampling.

- Dividing all cells into more groups with smallest possible group size is optimal in terms of lowering the total variation from perturbation.
Future Research

- $\xi < \frac{1}{3}$
- Different form of block transition matrix
- Sampling weights
- Other partitioning criteria
- Variation on the joint distribution between key and non-key variables
Thank You