A PASSAGE THROUGH BROBDINGNAG:

René Descartes and the Differential Calculus

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Abstract

On page 416 of volume I of *The Correspondence of Isaac Newton* edited by H. W. Turnbull et al, there appears a letter from Isaac Newton (1642-1726) to Robert Hooke (1635-1703) dated February 5, 1675 with a rather inconspicuous line, only to become one of the most often quoted statements in the history of mathematics:

If I have seen further it is by standing on ye shoulders of Giants.

Abstract

It was indeed the works of these giants – François Viète (1540-1603), William Oughtred (1575-1660), René Descartes (1596-1650), John Wallis (1616-1703), and Isaac Barrow (1630-1677) that helped Newton attain the mathematical maturity that would culminate in the complete development of the Fundamental Theorem of Calculus.

Abstract

In this paper we will look into the work of one of these giants - the "calculus" of Descartes, and see how it pertains to Newtonian calculus.

INTRODUCTION

- René Descartes (1596-1650)
- Born to moderately wealthy parents in La Haye, France.
- Received an excellent formal and traditional education at the Jesuit College of La Flèche.
- Eventually, he became apprehensive of the theories so many of his contemporaries and teachers professed and began to question the kind of knowledge that was being imparted to him.

INTRODUCTION

INTRODUCTION

Tired of what he felt to be fruitless intellectual discussions that seemed to be authoritarian and doctrinaire in nature, put aside his studies and wanted to take up a career in the military.

It was as a soldier in the army of the Duke of Bavaria, Descartes, found himself on the night of November 10, 1619, "shut up alone in a stove-heated room", and paradoxically enough, scientific modernism began with a set of three consecutive dreams that transpired on that night.

INTRODUCTION

For a period of several days prior to that date, Descartes had been feeling a “steady rise of temperature in his head” (Jaki 1978, 65): How could he establish a foundation for all knowledge so that it might have the same unity and certainty as mathematics? As a matter of fact, he was to write later in the famous *Discourse*

INTRODUCTION

Most of all I was delighted with Mathematics because of the certainty of its demonstrations and the evidence of its reasoning; but I did not yet understand its true use, and, believing that it was of service only in the mechanical arts, I was astonished that, seeing how firm and solid was its basis, no loftier edifice had been reared thereupon (Hutchins 1952, 43).

INTRODUCTION

During the dream Descartes saw before him two books. One was a dictionary, which appeared to him to be of little interest and use. The other was a compilation of poetry titled *Corpus Poetarum* in which there appeared to be a union of philosophy with wisdom.

INTRODUCTION

The way in which Descartes interpreted this dream set the stage for the rest of his life-long philosophical endeavors. For Descartes, the dictionary stood merely for scientific facts gathered together in a sterile, dry, disconnected, and unimaginative way, whereas the collection of poems symbolized the union of philosophy with wisdom.

INTRODUCTION

He became convinced that mathematics was the key to the secrets of nature. All of the sciences, he believed, were interconnected by mathematical links. The dreams showed Descartes that the entire universe was a great, harmonious, and mathematically designed machine.

INTRODUCTION

In 1628 he settled in Amsterdam and devoted the next twenty years of his life to critical and profound scrutiny of the nature of truth, the existence of God, the physical structure of the universe, and the study of mathematics.

In particular, from 1629 to 1633 he built up a cosmological theory, *the theory of vortices*, to explain all natural phenomena.

INTRODUCTION

- The vortex theory dominated seventeenth century thought until it was ultimately displaced by the cosmologies of Galileo and Newton.
- Descartes defined a *vortex* as a large circling band of material particles. The vortex theory attempted to explain celestial motion, especially orbits of planets by situating them in these vortices.
- But since the theory was a heliocentric one, he delayed its publication.

INTRODUCTION

This famous book, *Le Monde*, where he introduced his theory of vortices was published in 1664, 14 years after his death, and made him as dominant a figure in cosmology as he had been in mathematics and philosophy.

In 1648 Descartes was invited to Sweden by Queen Christina. He left the comfort of his home to tutor a queen whose work day started at 5 a.m. in an icy cold library. Eventually, he caught cold and died in 1650.

CARTESIAN PHILOSOPHY

- Descartes' philosophy, a philosophy of rationalism – knowledge comes from the intellect not from the senses – put him at odds with Aristotelian and consequently with Catholic teachings.
- All mental content could be reduced to three ideas that are innate and cannot be gained by experience of the external world: the idea one has of oneself, the idea one has of God, and the idea one has of materiality.
- It is from these primary and irreducible ideas one could derive, through the intellectual process of *deduction,* all the content of human science and wisdom. (*Discours de la méthode pour bien conduire sa raison et chercher la vérité dans les sciences 1667)*

CARTESIAN PHILOSOPHY

Dissatisfaction with dogmatically pronounced tenets, and establishment of the critical method of philosophy based on questioning everything led him to the following conclusions:

I think, therefore I am (*Cogito ergo sum* or *Je pense, donc je suis*)

Each phenomenon must have a cause

An effect cannot be greater than the cause

The mind within itself has the ideas of perfection, space, time, and motion

CARTESIAN PHYSICS

Time and space are existential forms of real world and matter is its substance. At a given moment in time a given piece of matter occupies a given portion of space. Thus, motion connects these three ingredients intimately. Descartes wanted to construct all natural occurrences from these basic concepts and thus reduce them to motion (*Principia Philosophae* Part II, 1644)

CARTESIAN PHYSICS

Motion was defined as *"nothing more than the action by which any body passes from one place to another, from the vicinity of those bodies, which are immediately contiguous to it and are viewed as if at rest, to the vicinity of others."* (*Principia* II, 24), which is too circular to be a scientifically acceptable definition – the word "motion" is not defined, it is simply replaced by the word "passes."

CARTESIAN PHYSICS

According to Descartes there were two causes of motion: the *general cause* governing all motion that existed in the world, and the *particular cause* dealing with the parts of matter acquiring motions that they did not have before. The general cause was God. The particular cause was subject to some laws, which he specified in *Le Monde as* the three laws of motion (*Descartes' Laws of Motion*), which served as a prototype to Newton's laws of motion:

CARTESIAN PHYSICS

Law of Persistence. Each part of matter, in particular, continues always to be in the same state, as long as an encounter with others does not constrain it to change (*Le Monde* 7, AT 11:38).

Law of Direction. When a body moves, even though its movement occurs most often in a curved line and though it cannot even make any motion that is not in some way circular […], still each of its parts in particular tends always to continue its own movement in a straight line. And so their action, that is, their inclination to move, is different from their movement (ibid, 7, AT 11:44).

The third law (ibid, 7, AT 11:41):
When a body impels another, it cannot give it any motion without losing at the same time the same amount of its own motion; nor take from it any, without augmenting its own by the same amount.

CARTESIAN GEOMETRY AND CALCULUS

The *Discours de la méthode* had three appendices: *La Dioptrique*, *Les Météores*, and *La Géométrie*. (1637) To show what his new method can accomplish outside of philosophy, Descartes had applied it to geometry and the outcome was *La Géométrie,* a short tract included with the anonymously published *Discourse on Method*.

CARTESIAN GEOMETRY AND CALCULUS

- Details a groundbreaking program for geometrical problem-solving—what he refers to as a "geometrical calculus" (*calcul géométrique*)—that rests on a distinctive approach to the relationship between algebra and geometry.
- Offers innovative algebraic techniques for analyzing geometrical problems, a novel way of understanding the connection between a curve's construction and its algebraic equation, and an algebraic classification of curves that is based on the degree of the equations used to represent these curves.

CARTESIAN GEOMETRY AND CALCULUS

La Géometrie was composed of three books:

1. *Problems Which Can Be Constructed by Means of Circles and Straight Lines Only*
2. *On the Nature of Curved Lines* - devoted mainly to applying algebra to geometry, i.e., to analytic geometry.
3. This part was on the theory of equations. Two important algebraic conventions were introduced here: using the last letters of the alphabet x, y, z as unknowns and the first letters of the alphabet as constant parameters and interpreting a quantity squared as an area, and a quantity cubed as a volume.

CARTESIAN GEOMETRY AND CALCULUS

The fundamental insight on which Descartes based his theory was described in the opening sentence of *La Géometrie*:

Any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction (Hutchins 1952, 295).

In other words, geometric problems could now be solved by algebraic means.

CARTESIAN GEOMETRY AND CALCULUS

Generalizations of Pappus' three-line problem and four-line problem. Descartes set out to show algebraically that the locus was a conic section, as stated without proof by Pappus, and the result was the birth of coordinate (analytic) geometry.

Descartes never used the word coordinate. The term was later introduced by Leibniz.

CARTESIAN GEOMETRY AND CALCULUS

Most importantly, he suggested a method for finding the equation of the tangent line to a curve (actually his method was for constructing the normal) in Book II *La Géometrie, On the Nature of Curved Lines*.

He would start with the equation of the curve and the location of the point *P*, at which the tangent was to be drawn, on the curve. He would then, using algebra, find the equation of the circle passing through the point *P* with center on the *x*-axis that had the same tangent line. Then he would construct the line tangent to the circle using a compass and a straightedge.

CARTESIAN GEOMETRY AND CALCULUS

More formally, suppose we have a curve $f(x, y) = 0$ and we want to draw the normal line at the point $P(x_0, y_0)$. Suppose moreover that this normal line intersects the x-axis at the point $Q(x_1, 0)$. Then the equation of the circle with center at Q and passing through the point P is

$$(x - x_1)^2 + y^2 = (x_0 - x_1)^2 + {y_0}^2$$

Now, eliminating the y variable between this equation and the equation $f(x, y) = 0$, gives us a relation involving only x and x_1, say, $g(x, x_1)$.

CARTESIAN GEOMETRY AND CALCULUS

Although in general the circle will intersect the curve in two points, since PQ is the normal, these two points will coincide and the circle will become tangent to the curve.

Thus, all one needs to do is to impose the condition that will make the equation $g(x, x_1) = 0$ have double roots, and use that to determine the value of x_1.

CARTESIAN GEOMETRY AND CALCULUS

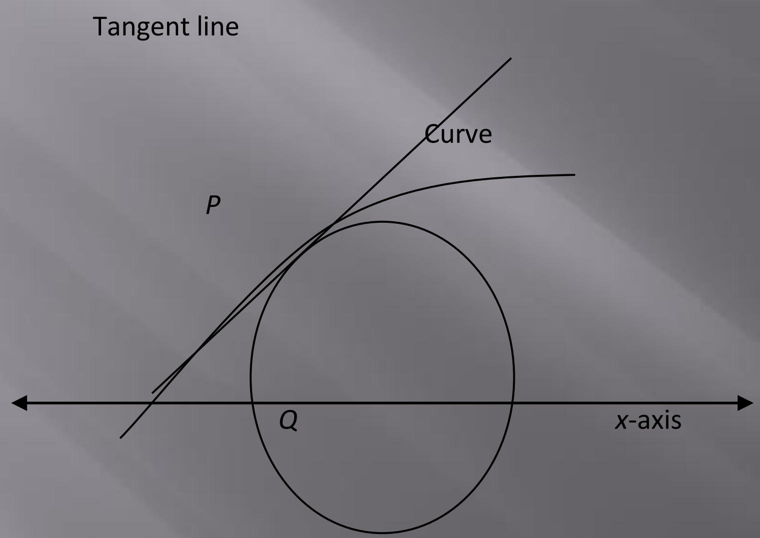

Figure 4.8 Descartes's Construction of the Tangent Line to an Arbitrary Curve at a Given Point

CARTESIAN GEOMETRY AND CALCULUS

To illustrate the point, let us determine the tangent to the parabola $y^2 = 2x$
at the point (8, 4). The equation of the circle with center at and passing through the point (8, 4) is

$$(x - x_1)^2 + y^2 = (8 - x_1)^2 + 16$$

Since $y^2 = 2x$, we get

$$x^2 + (2 - 2x_1)x + (16x_1 - 80) = 0$$

CARTESIAN GEOMETRY AND CALCULUS

Now in order for a quadratic to have double roots it must be of the form

$$x^2 + 2tx + t^2 = 0$$

Thus,

$$1 - x_1 = t$$

and

$$16x_1 - 80 = t^2$$

implying $x_1 = 9$. So the normal is the line that passes through the points (9,0) and (8, 4). So, the equation of the normal is $y = -4x + 36$ and that of the tangent is $y = {}^1/_4\, x + 2$