The Celestial Element Method

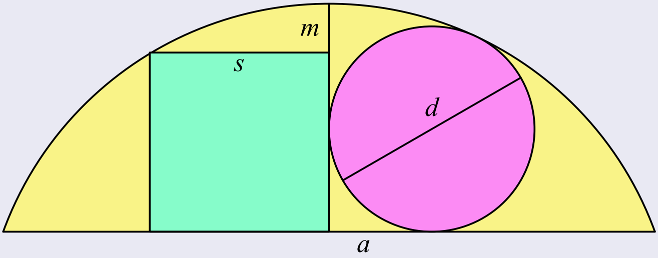
Finding Polynomial Roots in 18th Century Japan

David Clark

Randolph-Macon College

MAA Section Meeting
November 2, 2013

Gion Shrine Problem (c. 1749)



Consider the circular segment above: the cord has length a, and its perpendicular bisector has length m.

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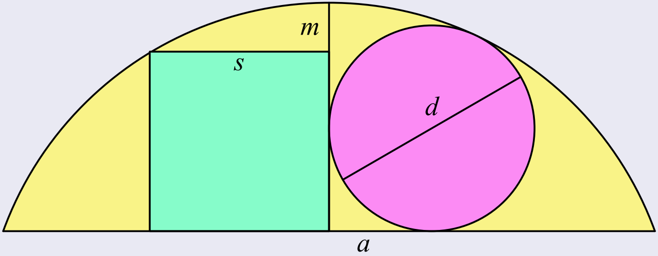
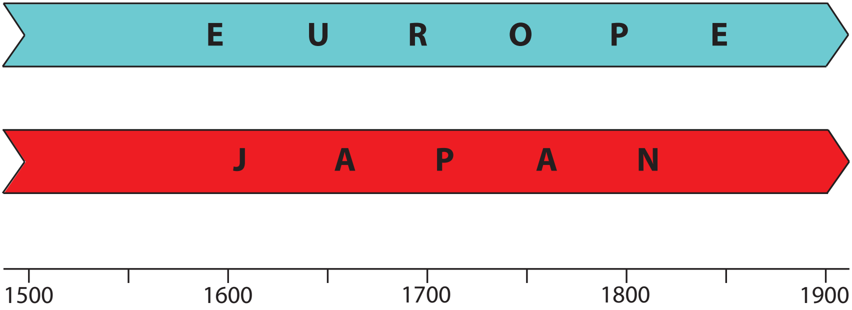


Consider the circular segment above: the cord has length a, and its perpendicular bisector has length m. Given

$$p = a + m + s + d \quad \textit{and} \quad q = \frac{m}{a} + \frac{d}{m} + \frac{s}{d},$$

find a, m, s, and d.

18th century Japan



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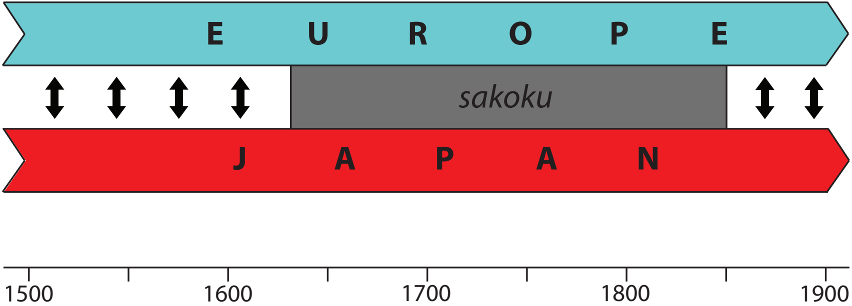

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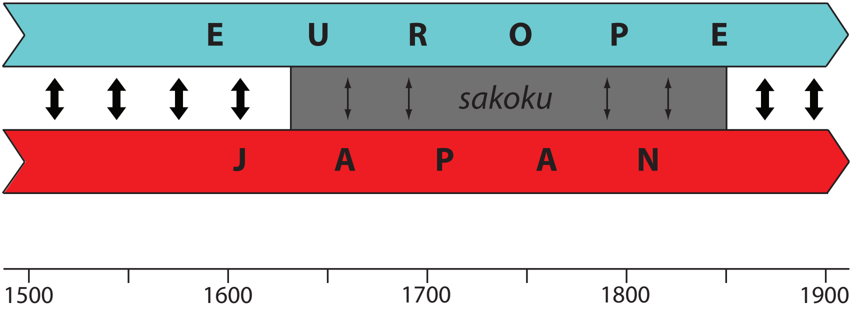

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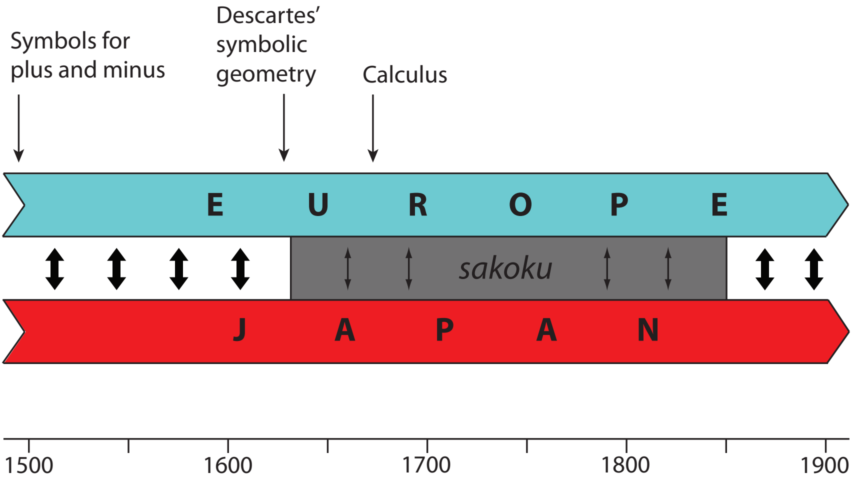

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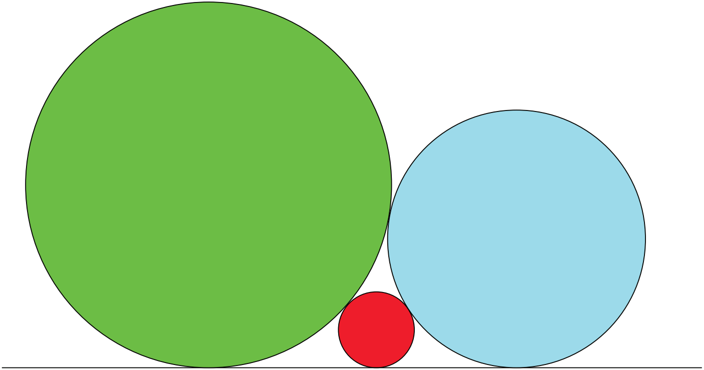

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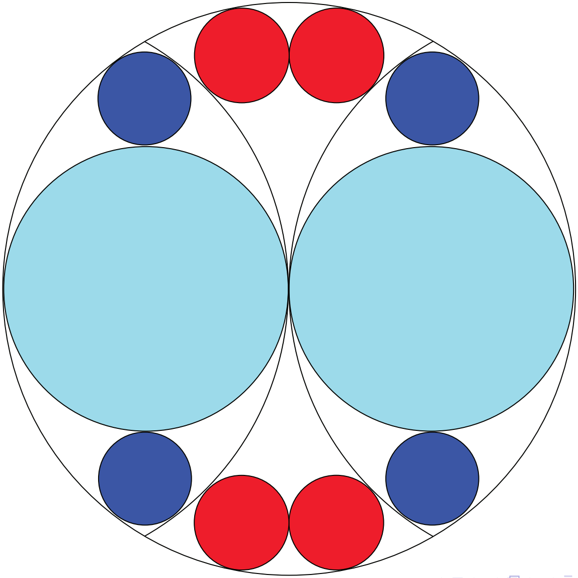

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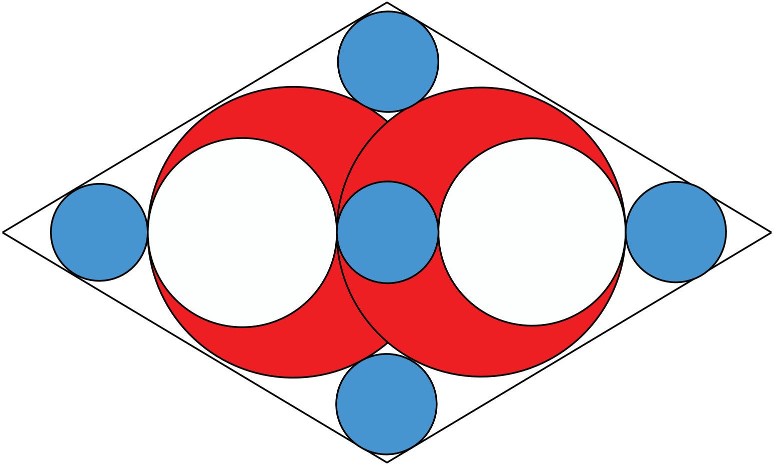

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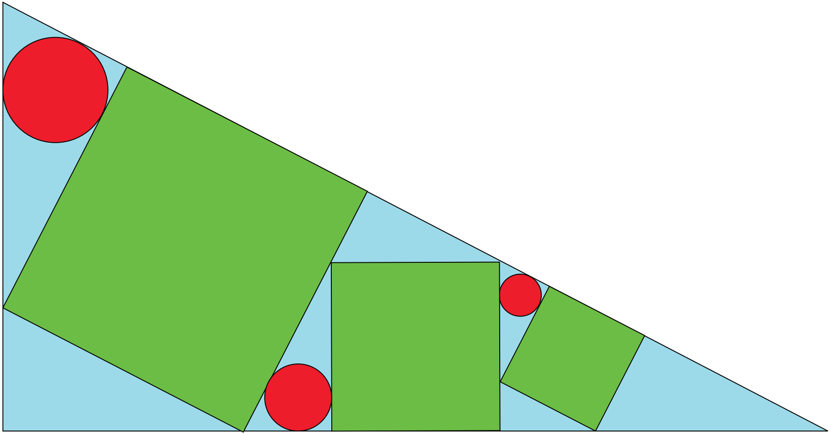

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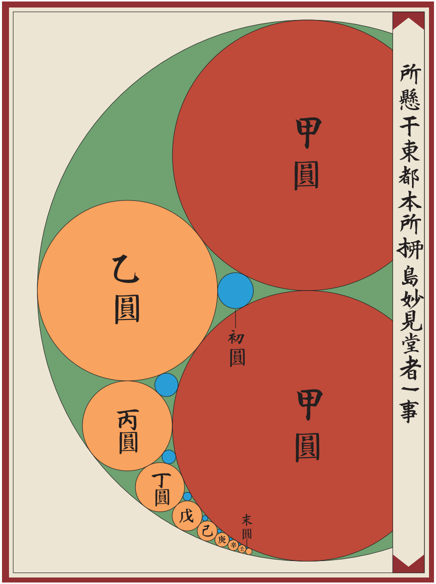

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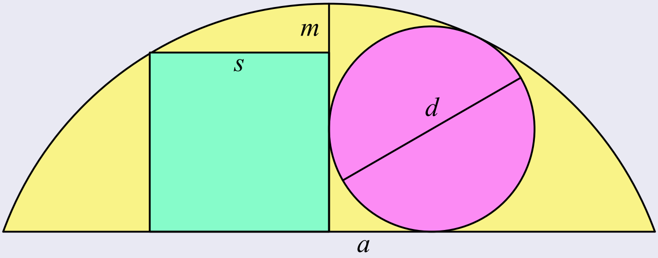

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find a, m, s, and d.

East vs. West

Numerical approach to polynomial equations:

- West: Newton's method, Horner's method

East vs. West

Numerical approach to polynomial equations:

- West: Newton's method, Horner's method
- East: Method of the celestial element


文久元年

Swanpan board

万	千	百	十	一	分	
						商
						実
						方
						廉
						偶
						三

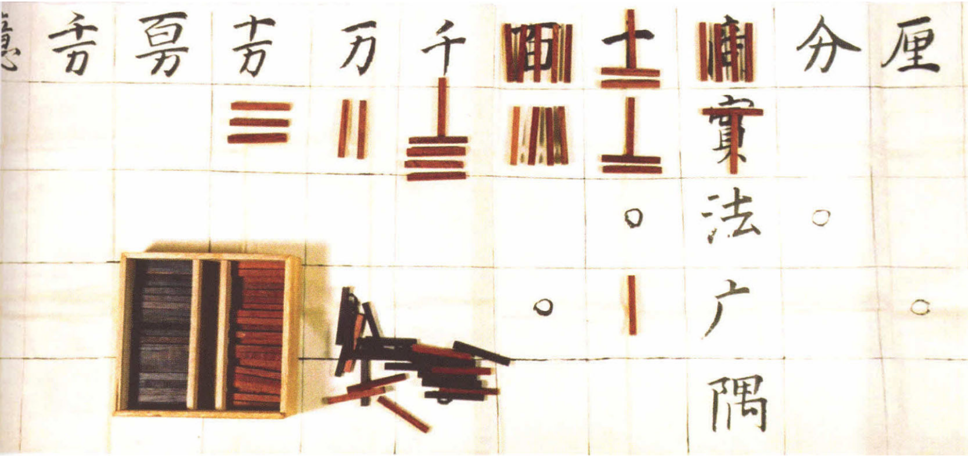
Swanpan board

10000 万	1000 千	100 百	10 十	1 一	0.1 分	
						商
						実
						方
						廉
						偶
						三

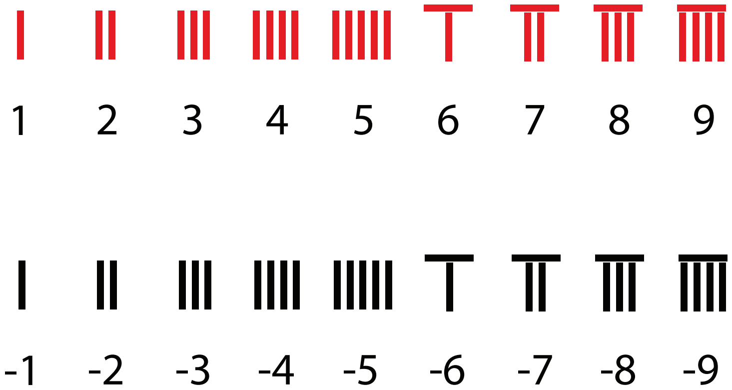
Swanpan board

10000 万	1000 千	100 百	10 十	1 一	0.1 分	
						商 sho
						実 jitsu
						方 ho
						廉 ren
						偶 gu
						三 san

Sangi computing rods



Sangi computing rods



Sangi computing rods

"Method of the celestial element" from the *Tengen Shinan* (1698):

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$

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万	千	百	十	一	
					商
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					方
					廉
					偶
					三

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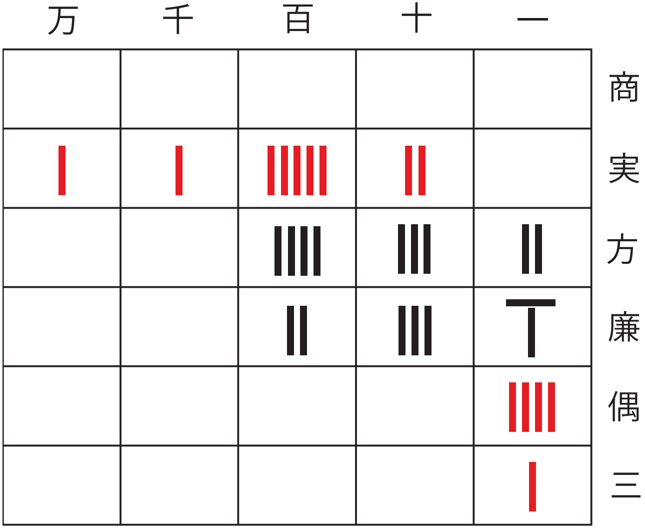

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$

10000	1000	100	10	1	
					sho
1	1	5	2		jitsu
		4	3	2	ho
		2	3	6	ren
				4	gu
				1	san

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10000	1000	100	10	1	
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1	1	5	2		jitsu
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10000	1000	100	10	1	
			1		sho
	2	4			jitsu
		4	8		ho
1	4	4			ren
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1					san

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1 →					san

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10000	1000	100	10	1	
			1		sho
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Horner's method

Solve: $p(x) = p_0 + p_1 x + \ldots + p_n x^n = 0$

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Step 1: Make an initial guess, $x = E_1$.

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Step 1: Make an initial guess, $x = E_1$. Then write

$$p(x) = a_0 + a_1(x - E_1) + a_2(x - E_1)^2 + \ldots + a_n(x - E_1)^n.$$

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Step 2: Substitute $y = x - E_1$, so we have

$$g(y) = a_0 + a_1 y + a_2 y^2 + \ldots + a_n y^n = 0.$$

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- If this guess is correct, then $E_2 = x - E_1$, so $x = E_1 + E_2$.

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- If this guess is correct, then $E_2 = x - E_1$, so $x = E_1 + E_2$.
- If not, repeat **Step 2** with the substitution $z = y - E_2$.

Horner's method

Example:

$$p(x) = 11520 - 432x - 236x^2 + 4x^3 + x^4$$

Horner's method

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\begin{aligned}
p(x) &= 11520 - 432x - 236x^2 + 4x^3 + x^4 \\
&= -2400 + 48(x-10) + 484(x-10)^2 + 44(x-10)^3 + (x-10)^4
\end{aligned}
$$

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Now let $y = x - 10$, and define

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Since $y = 2$ is a root of g, we have that $2 = x - 10$.

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Thus $x = 12$ is a root of p.

Long division vs. synthetic division

$$
\begin{array}{rrrrrrl}
 & & x^3 & +14x^2 & -96x & -1392 & \textbf{r} \; -2400 \\
\cline{2-6}
x \; -10 \;) & x^4 & +4x^3 & -236x^2 & -432x & +11520 & \\
 & x^4 & -10x^3 & & & & \\
\cline{2-4}
 & & 14x^3 & -236x^2 & & & \\
 & & 14x^3 & -140x^2 & & & \\
\cline{3-5}
 & & & -96x^2 & -432x & & \\
 & & & -96x^2 & +960x & & \\
\cline{4-6}
 & & & & -1392x & +11520 & \\
 & & & & -1392x & +13920 & \\
\cline{5-6}
 & & & & & -2400 & \\
\end{array}
$$

Long division vs. synthetic division

10	1	4	−236	−432	11520

Long division vs. synthetic division

10	1	4	-236	-432	11520
	1				

Long division vs. synthetic division

10	1	4	−236	−432	11520
		10			
	1				

Long division vs. synthetic division

10	1	4	-236	-432	11520
		10			
	1	14			

Long division vs. synthetic division

10	1	4	−236	−432	11520
		10	140		
	1	14			

Long division vs. synthetic division

10	1	4	−236	−432	11520
		10	140		
	1	14	− 96		

Long division vs. synthetic division

10	1	4	−236	−432	11520
		10	140	− 960	
	1	14	− 96		

Long division vs. synthetic division

10	1	4	−236	−432	11520
		10	140	− 960	
	1	14	− 96	− 1392	

Long division vs. synthetic division

10	1	4	−236	−432	11520
		10	140	− 960	− 13920
	1	14	− 96	− 1392	

Long division vs. synthetic division

10	1	4	−236	−432	11520
		10	140	− 960	− 13920
	1	14	− 96	− 1392	− 2400

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10	1	4	−236	−432	11520
		10	140	− 960	− 13920
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Long division vs. synthetic division

10	1	4	−236	−432	11520
		10	140	− 960	− 13920
10	1	14	− 96	− 1392	− 2400
	1				

Long division vs. synthetic division

10	1	4	−236	−432	11520
		10	140	− 960	− 13920
10	1	14	− 96	− 1392	− 2400
		10	240	1440	
	1	24	144	48	

Long division vs. synthetic division

10	1	4	−236	−432	11520
		10	140	− 960	− 13920
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		10	240	1440	
10	1	24	144	48	

Long division vs. synthetic division

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		10	240	1440	
10	1	24	144	48	
		10	340		
	1	34	484		

Long division vs. synthetic division

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Long division vs. synthetic division

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		10	240	1440	
10	1	24	144	48	
		10	340		
10	1	34	484		
		10			
	1	44			

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10000	1000	100	10	1	
					sho
1	1	5	2		jitsu
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	2	4			jitsu
1	3	9	2		ho
	9	6			ren
1	4				gu
1					san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$

10000	1000	100	10	1	
			1		sho
	2	4			jitsu
1	3	9	2		ho
	9	6			ren
2	4				gu
1					san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$

10000	1000	100	10	1	
			1		sho
	2	4			jitsu
1	3	9	2		ho
1	4	4			ren
2	4				gu
1					san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$

10000	1000	100	10	1	
			1		sho
	2	4			jitsu
		4	8		ho
1	4	4			ren
2	4				gu
1					san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$

10000	1000	100	10	1	
			1		sho
	2	4			jitsu
		4	8		ho
1	4	4			ren
3	4				gu
1					san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$

10000	1000	100	10	1	
			1		sho
	2	4			jitsu
		4	8		ho
4	8	4			ren
3	4				gu
1					san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$

10000	1000	100	10	1	
			1		sho
	2	4			jitsu
		4	8		ho
4	8	4			ren
4	4				gu
1					san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$

10000	1000	100	10	1	
			1		sho
	2	4			jitsu
		4	8		ho
4	8	4			ren
4	4				gu
1					san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$
$$-2400 + 48y + 484y^2 + 44y^3 + y^4 = 0$$

10000	1000	100	10	1	
			1		sho
	2	4			jitsu
			4	8	ho
		4	8	4	ren
			4	4	gu
				1	san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$
$$-2400 + 48y + 484y^2 + 44y^3 + y^4 = 0$$

10000	1000	100	10	1	
			1	2	sho
	2	4			jitsu
			4	8	ho
		4	8	4	ren
			4	4	gu
				1	san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$
$$-2400 + 48y + 484y^2 + 44y^3 + y^4 = 0$$

10000	1000	100	10	1	
			1	2	sho
	2	4			jitsu
			4	8	ho
		4	8	4	ren
			4	6	gu
				1	san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$
$$-2400 + 48y + 484y^2 + 44y^3 + y^4 = 0$$

10000	1000	100	10	1	
			1	2	sho
	2	4			jitsu
			4	8	ho
		5	7	6	ren
			4	6	gu
				1	san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$
$$-2400 + 48y + 484y^2 + 44y^3 + y^4 = 0$$

10000	1000	100	10	1	
			1	2	sho
	2	4			jitsu
	1	2			ho
		5	7	6	ren
			4	6	gu
				1	san

$$11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$$
$$-2400 + 48y + 484y^2 + 44y^3 + y^4 = 0$$

10000	1000	100	10	1	
			1	2	sho
					jitsu
	1	2			ho
		5	7	6	ren
			4	6	gu
				1	san

Example:

$$\begin{aligned} p(x) &= 11520 - 432x - 236x^2 + 4x^3 + x^4 \\ &= -2400 + 48(x-10) + 484(x-10)^2 + 44(x-10)^3 + (x-10)^4 \end{aligned}$$

Now let $y = x - 10$, and define

$$g(y) = -2400 + 48y + 484y^2 + 44y^3 + y^4.$$

Since $y = 2$ is a root of g, we have that $2 = x - 10$.

Thus $x = 12$ is a root of p.

Theorem

Let $p(x) = p_0 + p_1 x + \ldots + p_n x^n$ *and say*

$$\begin{aligned} p(x) =: q_0(x) &= q_1(x)(x-a) + r_0 \\ q_1(x) &= q_2(x)(x-a) + r_1 \\ q_2(x) &= q_3(x)(x-a) + r_2 \\ \vdots &= \vdots \\ q_{n-1}(x) &= q_n(x)(x-a) + r_{n-1} \end{aligned}$$

Theorem

Let $p(x) = p_0 + p_1 x + \ldots + p_n x^n$ *and say*

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Theorem

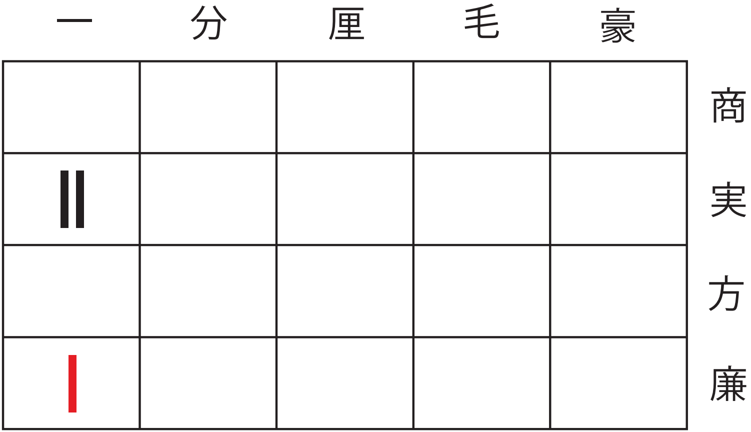
Let $p(x) = p_0 + p_1 x + \ldots + p_n x^n$ *and say*

$$\begin{aligned} p(x) =: q_0(x) &= q_1(x)(x-a) + r_0 \\ q_1(x) &= q_2(x)(x-a) + r_1 \\ q_2(x) &= q_3(x)(x-a) + r_2 \\ \vdots &= \vdots \\ q_{n-1}(x) &= q_n(x)(x-a) + r_{n-1} \\ q_n(x) &= p_n = r_n. \end{aligned}$$

Then we can write

$$p(x) = r_0 + r_1(x-a) + r_2(x-a)^2 + \ldots + r_{n-1}(x-a)^{n-1} + r_n(x-a)^n.$$

$$-2 + x^2 = 0$$



$$-2 + x^2 = 0$$

1	0.1	0.01	0.001	0.0001	
					sho
2					jitsu
					ho
1					ren

$$-2 + x^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1					sho
2					jitsu
					ho
1					ren

$$-2 + x^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1					sho
2					jitsu
1					ho
1					ren

$$-2 + x^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1					sho
1					jitsu
1					ho
1					ren

$$-2 + x^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1					sho
1					jitsu
2					ho
1					ren

$$-2 + x^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1					sho
1					jitsu
2 →					ho
1 → →					ren

$$-2 + x^2 = 0$$
$$-1 + 0.2y + 0.01y^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1					sho
1					jitsu
	2				ho
		1			ren

$$-2 + x^2 = 0$$

$$-1 + 0.2y + 0.01y^2 = 0 \Leftrightarrow -100 + 20y + y^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1					sho
1					jitsu
	2				ho
		1			ren

$$-2 + x^2 = 0$$

$$-1 + 0.2y + 0.01y^2 = 0 \Leftrightarrow -100 + 20y + y^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1	4				sho
1					jitsu
	2				ho
		1			ren

$$-2 + x^2 = 0$$

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1	0.1	0.01	0.001	0.0001	
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1	4				sho
		4			jitsu
	2	4			ho
		1			ren

$$-2 + x^2 = 0$$

$$-1 + 0.2y + 0.01y^2 = 0 \Leftrightarrow -100 + 20y + y^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1	4				sho
		4			jitsu
	2	8			ho
		1			ren

$$-2 + x^2 = 0$$

$$-1 + 0.2y + 0.01y^2 = 0 \Leftrightarrow -100 + 20y + y^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1	4				sho
		4			jitsu
	2	8 →			ho
		1 →			ren

$$-2 + x^2 = 0$$

$$-1 + 0.2y + 0.01y^2 = 0 \Leftrightarrow -100 + 20y + y^2 = 0$$

$$-0.04 + 0.028z + 0.0001z^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1	4				sho
		4			jitsu
		2	8		ho
				1	ren

$$-2 + x^2 = 0$$
$$-1 + 0.2y + 0.01y^2 = 0 \Leftrightarrow -100 + 20y + y^2 = 0$$
$$-0.04 + 0.028z + 0.0001z^2 = 0 \Leftrightarrow -400 + 280z + z^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1	4				sho
		4			jitsu
		2	8		ho
				1	ren

$$-2 + x^2 = 0$$

$$-1 + 0.2y + 0.01y^2 = 0 \Leftrightarrow -100 + 20y + y^2 = 0$$

$$-0.04 + 0.028z + 0.0001z^2 = 0 \Leftrightarrow -400 + 280z + z^2 = 0$$

1	0.1	0.01	0.001	0.0001	
1	4	1			sho
		4			jitsu
		2	8		ho
				1	ren

East vs. West

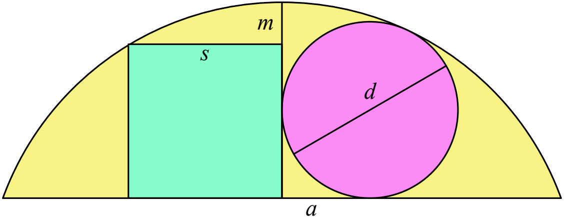
Horner's method was named after William Horner (1786 - 1837) . . .

East vs. West

Horner's method was named after William Horner (1786 - 1837) . . .

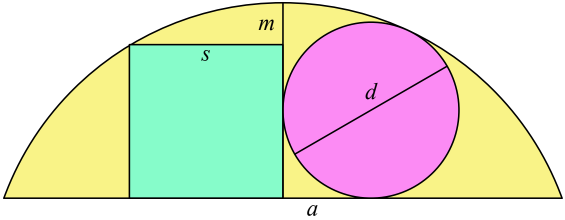
. . . but the method of the celestial element was developed in 13th century China!

Gion Shrine Problem



Given $p = a + m + s + d$ and $q = \dfrac{m}{a} + \dfrac{d}{m} + \dfrac{s}{d}$, find a, m, s, and d.

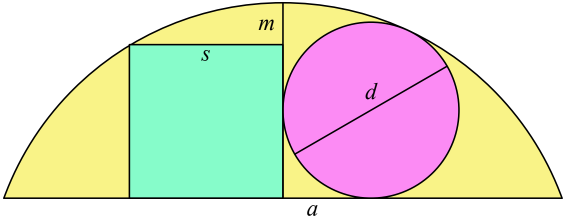
Gion Shrine Problem



Given $p = a + m + s + d$ and $q = \dfrac{m}{a} + \dfrac{d}{m} + \dfrac{s}{d}$, find a, m, s, and d.

- 1749: Tsuda Nobuhisa – polynomial of degree 1024

Gion Shrine Problem



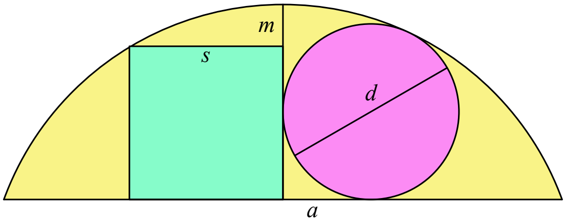
Given $p = a + m + s + d$ and $q = \frac{m}{a} + \frac{d}{m} + \frac{s}{d}$, find a, m, s, and d.

- 1749: Tsuda Nobuhisa – polynomial of degree 1024
- 17??: Nakata – polynomial of degree 46

Gion Shrine Problem

万	千	百	十	一	分

Gion Shrine Problem



Given $p = a + m + s + d$ and $q = \frac{m}{a} + \frac{d}{m} + \frac{s}{d}$, find a, m, s, and d.

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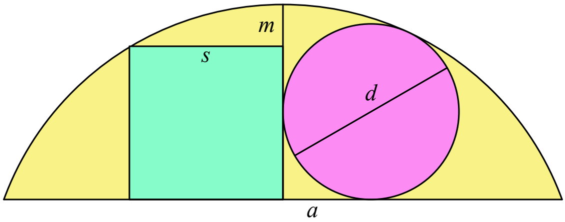
Gion Shrine Problem



Given $p = a + m + s + d$ and $q = \frac{m}{a} + \frac{d}{m} + \frac{s}{d}$, find a, m, s, and d.

- 1749: Tsuda Nobuhisa – polynomial of degree 1024
- 17??: Nakata – polynomial of degree 46
- 1774: Ajima Naonobu – polynomial of degree 10

Note: some images in this presentation first appeared in

- *Sacred Mathematics: Japanese Temple Geometry* by Fukagawa Hidetoshi and Tony Rothman (Princeton, 2008)
- "Japanese Temple Geometry," by Tony Rothman, *Scientific American*, May 1998.