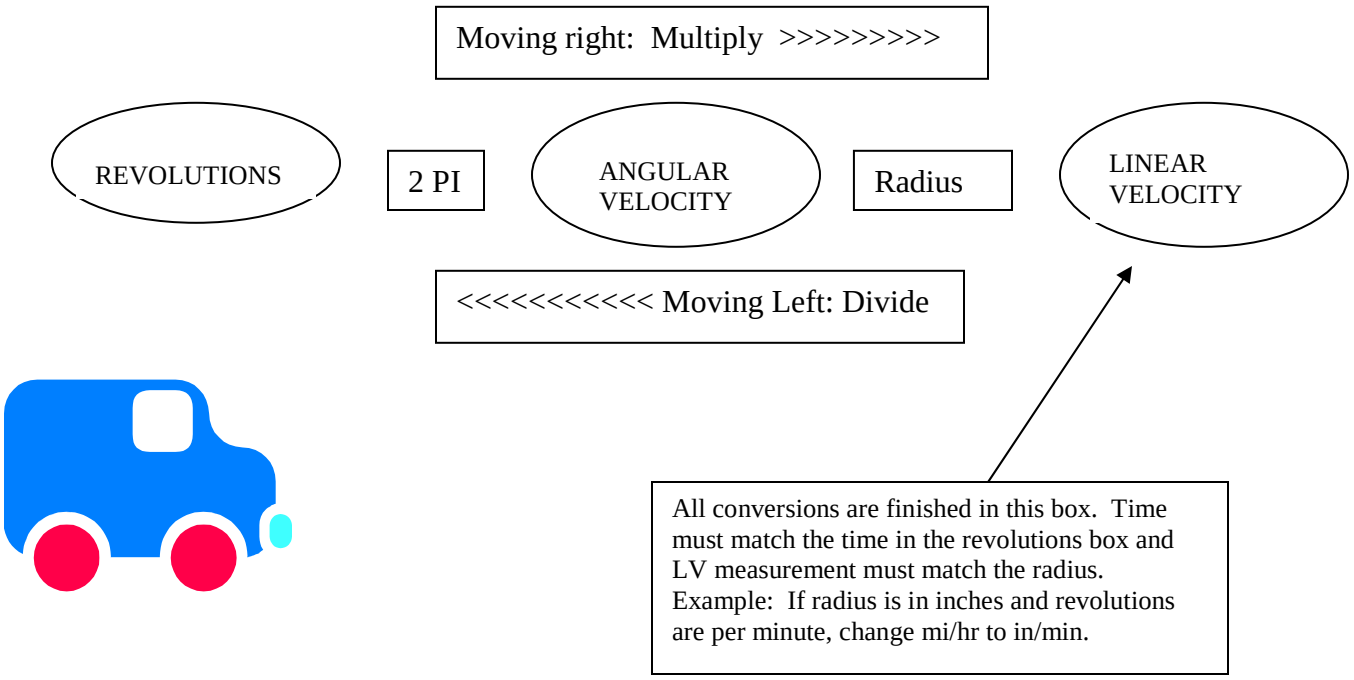
The Wheels on the Bus … or What Goes Around….

Q: Why does the angle always have to be in radians?
A: It just works.
Q: But why?
A: The math police say so.
Q: But w-h-y-y-y-y?

A: Well, if you really want to know, here's a picture. You can work any kind of revolution, angular velocity, or linear velocity problem this way. And if you put two diagrams together, you can solve wheel problems, knowing their linear velocities are the same.



Q: Can you give me a hint about why the angle has to be in radians?
A: Focus in on the word "radius" in the diagram. Pretend you are going 60 mi/hr (Linear Velocity) in a car with a tire with radius 15 in. You want to know Angular Velocity. Change your 60 mi/hr to 63360 in/min, so inches matches your radius and minutes

matches your revolutions (rpm). You have to move left in the picture, and when you move left, you divide. So, divide 63360 in /min by 15 IN. PER RADIUS.

$$\frac{\dfrac{63360 \text{ in}}{1 \text{ minute}}}{\dfrac{15 \text{ in}}{1 \text{ radius}}} = \frac{4224 \text{ radius in}}{1 \text{ minute in}}$$

$$\frac{4224 \text{ radius}}{1 \text{ minute}}$$

$$\frac{\dfrac{4224 \text{ radius}}{1 \text{ minute}}}{\dfrac{1 \text{ radius}}{1 \text{ radian}}} = \frac{4224 \text{ radius radians}}{1 \text{ radius minute}}$$

$$\frac{4224 \text{ radians}}{1 \text{ minute}}$$ This is your angular velocity. ω

Only radians have a 1 to 1 ratio with the radius. It's all based on the definition of a radian. Radians work. Degrees don't.

Later:
Q: Yes, I see why it works only when the angle is in radians!
Then I can divide (4224 radians /1 minute) by (2π radians / revolution) and get 672.27 revolutions per minute. But why does it have to be π?
A: The circumference is $2\pi r$.
Q: But, why is it the number π?
A: The math police say so.
Q: But why?
A: That's just how it works.
Q. But w-h-y-y-y-y?