UNDERGRADUATE RESEARCH: SOME CONCLUSIONS

Kenneth O. May, University of Toronto

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Original contributions to mathematics by juveniles have a very long tradition. Indeed precocity and early involvement in creative work appear to be characteristic of practically all of the most productive mathematicians of the past, and there seems no reason to think that mathematics has changed enough to make this any less likely in the future.

The term "undergraduate research" has a somewhat different meaning. It refers to an educational activity designed to increase the output of creative mathematicians and scientists by giving young people the experience of original work in addition to the standard courses in which they "learn" what has been discovered by others. Undergraduate research activities may, and often do, lead to new publishable mathematical results, but this is not the goal. The activity may be considered successful if it enhances and accelerates the student's development as an original thinker. The important manifestations are an effort involving independent work and results that are original for the student, regardless of their newness to other mathematicians or significance for current mathematical research.

Undergraduate research as an educational activity, has a long history in the United States. (9, 10). Part of the tradition stems from the senior thesis required at many institutions from the nineteenth century and earlier. At Reed College, under the loving guidance of Griffin, the senior thesis became an occasion for much original work. (2, 3). Undoubtedly similar activities took place at other colleges, and this kind of activity shades off into similar pedagogical techniques not usually called undergraduate research. Examples are the "Moore method," contests, and the "discovery approach" to all levels. The inspiring teacher who stimulates his students to work on their own and solve challenging problems is promoting undergraduate research.

The recent development of undergraduate research programs is part of the general revival of interest in improving the teaching of mathematics at all levels. The new element has been a concious organized effort to stimulate activity with or without financial support from outside. Having been involved in experimentation in this area for a decade at Carleton College, I here offer a few conclusions based on my own experiences and those of others whose work came to my attention as editor of Delta-Epsilon and a participant in many discussions.(1, 5, 6).

1. Staff. It is essential to have a staff member A who is genuinely interested in undergraduates and enthusiastic about their work. Personal encouragement, interest, and enthusiasm are undoubtedly the most important means for motivating students. Mathematical originality and research activity are helpful but not essential, provided the staff member is intellectually alive and interested in mathematical ideas. It is also essential to have a staff member B who believes in the educational value of undergraduate research activities and who has sufficient influence to gain the necessary support from his colleagues and the administration. Supervision of undergraduate research takes time and energy. A small group of active students, possibly as few as three, is the equivalent of a course and should be so recognized. In the ideal department every member is both an A and a B, but a minimum is one of each with A = B.

II. Organization. Work has been carried out successfully with every imaginable organizational structure. (4, 6). Among these might be mentioned the following: A formal course for credit run like a seminar with students reporting on their individual or group research projects. Special honors courses and "problem" courses. A student run colloquium at which problems are discussed and work reported, but with no credit and only very loose faculty involvement. An apparently anarchic "atmosphere" in which it "just happens" that students are inspired to do supplementary original work for their professors, some of which leads to publication. It would be impossible to list the variations, but it is essential that some concious effort be extended by the faculty and students. "Anarchic" stituations are usually a result of a carefully nurtured tradition, and definite arrangements for academic credit are helpful.

III. Publications. It frequently happens that undergraduate papers are published in the professional journals. This is more common than is generally realised, because editors prefer not to call attention to the undergraduate origin of a paper. (I have been told that this preference arises from the fear of being deluged with undergraduate garbage.) However, such publication is not the important goal of an undergraduate research program. Student papers should not be judged in terms of their "publishability". On the other hand the very highest standards of clarity, exposition, and correctness should be demanded. Indeed one goal should be the eventual improvement of the presently very low standards of exposition in the mathematical community. A well written paper that reflects independent and original work on the part of the student, even if his results are not new, deserves praise and recognition. Moreover possible publication provides incentive, and actual publication stimulates further effort. For these reasons it is desirable to have arrangements for local publication. This can easily be accomplished if staff and students keep in mind that they are not trying to emulate existing mathematical journals but instead are providing an opportunity for immediate publication and modest circulation of

good student work. Publication dates need not be regular. Duplication of amateur typescript by ditto, mimeo, or multilith is adequate. The level can vary with the author's background. Reviews and news can be included. The work of such publication can be done largely by the student themselves and duplicating absorbed in the day-to-day operations of the department (1, 9).

IV. An ideal program? None exists, but a very good one would involve the following:

1. A student run colloquium

2. A student run publication

3. A student mathematics club that supports these activities and engages in others appropriate to the local situation.

4. A faculty member who keeps a friendly eye on things and helps as needed.

5. Some link of these activities with the curriculum.

Ideally, research-like activity should be part of every course (the quality and quantity varying with the context) and should be officially recognized by graduation honors, special courses, and the like. In short, undergraduate research should become a normal part of the curricular and extra-curricular educational process (8). Teachers should be judged in part by the degree to which they stimulate independent work. But this is not the case for many reasons, and special programs and projects should be designed both for their immediate productivity and to assist in creating the environment in which they will no longer be necessary.

V. A(n) (inter)national student publication? College students sometimes read professional magazines, especially the American Mathematical Monthly and occasionally the more specialized research journals. But these periodicals are not really addressed to students (many of them do not seem to be addressed to anyone!). It is not just a matter of level, but more one of tone, style, and point of view. A magazine addressed to college mathematics students might make an important contribution to their education. It could, from a student point of view, review books of particular interest to undergraduates, present expositions that would inform and stimulate, include news of student mathematical organizations (Pi Mu Epsilon, Kappa Mu Epsilon, clubs, etc.) and activities, provide an outlet for the very best undergraduate writing and a forum for student discussion, contain a problem department designed for students, --and much more that cannot be anticipated. For example, it might print news stories (including pictures) of the Putnam winners, various solutions of Putnam questions, etc. It could encourage inter-college student meetings and competitions,

Such a magazine might have its greatest individual impact where the student is "neglected" (at very small places with inadequate staff and at very large places with preoccupied staff). Its greatest value would be to reach and stimulate the undergraduate by putting him in communication with other students and mature mathematicians. The editorial staff should include students, though responsibility for continuity and management would best be assumed by the M.A.A. or other continuing professional organizations. (7).

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REFERENCES

1. Delta-Epsilon, a journal for undergraduate work, published at Carleton College from January 1960, with N.S.F. support 1961-1966. Volume 6 appeared in 1965-1966. Besides undergraduate papers, it contains reports on programs at various colleges. Backfiles are obtainable from the present editor, Prof. Roger Kirchner, Department of Mathematics, Carleton College, Northfield, Minnesota.

2. F. L. Griffin. "Undergraduate mathematical research," Amer. Math. Monthly, 49 (1942), 379-385.

3. F. L. Griffin. "Further experience with undergraduate mathematical research," Amer. Math. Monthly, 58, (1951), 322-325.

4. C. B. Lindquist. Honors programs for superior undergraduate mathematics students. OE-56015. Mathematics Programs Series, Divis. of Higher Ed., USDHEW, Office of Education. Washington, 1964.

5. Kenneth O. May. "Undergraduate research in mathematics," Amer. Math. Monthly, 55 (1948), 241-246.

6. Kenneth O. May and Seymour Schuster, editors. Undergraduate Research in Mathematics, Report of a conference held at Carleton College, June 19-23, 1961, with support of the National Science Foundation. Northfield, 1962. Out of print but copies were sent to all U.S. departments and libraries at the college level. Includes papers, examples, devices, case histories, topics, and bibliography.

7. Particle, a quarterly "by and for science students" published in Berkeley, California, has managed to keep going since 1960, but with considerable difficulty because of the lack of continued financial backing and scientific guidance.

8. H. E. Roscoe. "Original research as a means of education." Nature, October 23 and 30, 1873, 538-9 and 559-561. Roscoe, a chemist, makes a plea for research as part of education, sometimes in a quite quaint way. Summing up, he says that his aim has been to show that "if freedom of inquiry, independence of thought, disinterested and steadfast labor, habits of exhaustive and truthful observation, and of clear perception, are things to be desired as tending to the higher intellectual development of mankind, then original research ought to be included as one of the most valuable means of education." Poor Roscoe! Such appeals have secured equipment and leisure for research by professors, but education remains largely the memorization and regurgitation of the discoveries of others.

9. L. G. Simons. "Undergraduate publications in mathematics," Scripta Mathematica, 8 (1941), 165-175. Describes many local student mathematics journals, of which the oldest appears to be the proceedings of the University of Toronto Mathematical and Physical Society. (1882!). Simons cites an example of a phenomenon that appears to be quite common: the editor of a student publication turns up a few years later as a faculty advisor. It appears that one result of undergraduate research activities is to help produce teachers interested in students!.

10. E. R. Sleight. "Undergraduate research in Michigan," Amer. Math. Monthly, 48 (1941), 696-697.