The April Meeting of the Iowa Section

The 46th meeting of the Iowa Section of the Mathematical Association of America was held at Iowa Wesleyan College, Mt. Pleasant, Iowa, April 17, 1959. Professor E. N. Oberg, Chairman of the Section, presided. Total attendance was 50, including 23 members of the Association. Routine business was considered during the afternoon meeting.

The following officers were elected:

Chairman, Professor J. J. L. Hinrichsen, Iowa State College, Ames, Iowa

Vice-Chairman, Professor H. C. Trimble, Iowa State Teacher's College, Cedar Falls, Iowa

Secretary-Treasurer, Professor Earle L. Canfield, Drake University, Des Moines, Iowa

The following resolution was discussed:

> "In case the Iowa Sectional meeting coincides with the spring Society meeting, that the sectional meeting be scheduled for another week end."

No specific action was taken on the resolution and it was left for consideration by the officers for another year.

The following papers completed the program:

Friday morning, April 17, 1959.

Wedges: A discussion of partially ordered linear spaces, by Professor Mahlon M. Day, University of Illinois.
(By invitation).

The useful properties of a partial order relation ($\geq$) among elements of a linear space L can be characterized by geometric properties of the set P of elements $\geq$ 0. Examples were given. Fixed vectors and fixed directions of linear operators preserving order were discussed next. Monotone linear functionals, their monotone linear extensions, and the geometric interpretation of the theorems which are known were mentioned.

A necessary condition for the completeness of a family of probability measures, by Professor Robert V. Hogg, University of Iowa.

A family $\{F(x;\theta);\ \theta \in \Omega\}$ of distribution functions is complete if $\int_{-\infty}^{\infty} u(x)\,dF(x;\theta) = 0$, for all $\theta \in \Omega$ and any real $u(x)$ that is absolutely integrable with respect to $\{F\}$, implies that $u(x) \equiv 0$ almost everywhere $\{F\}$. Let $\Phi(t;\theta) \equiv \int_{-\infty}^{\infty} \exp(itx)\,dF(x;\theta)$ be the characteristic function of $F(x;\theta)$. A family $\{\Phi(t;\theta);\ \theta \in \Omega\}$ is c-complete if

$$\int_{-\infty}^{\infty} \left[\alpha(t) + i\,\beta(t)\right] \Phi(t;\theta)\,dt = 0$$

for all $\theta \in \Omega$ and any real $\alpha(t)$ and $\beta(t)$ that are absolutely integrable, implies that $\alpha(t) \equiv 0$ and $\beta(t) \equiv 0$ almost everywhere Lebesgue. A necessary condition that the family $\{F(x;\theta);\ \theta \in \Omega$ be complete is that the family $\{\Phi(t;\theta);\ \theta \in \Omega\}$ be c-complete.

Method of solving a class of mixed boundary value problems, by Professor Don Kirkham, Iowa State College.

It is shown, by solving two practical potential problem examples, how mixed boundary value functions in a class of problems may be developed, over a boundary, into an infinite series of harmonic functions, leading to the solution. The development is accomplished by introduction of an auxiliary infinite series valid over the portion of the boundary where the normal derivative is given. One of the examples is a two-dimensional one for which a solution can also be obtained by conformal transformations. The solution by the two methods agree. The other example involves Bessel functions. The method of this paper does not require the breaking up of the region in question into auxiliary spaces.

Waring's problem, Modulo p, and the representation symbol, by Sister M. Anne Cathleen Real, Marycrest College, presented by Nancy Ketelaar, introduced by the Chairman.

The representation symbol $\left[a, b, c\right]$ is the statement that an integer of n-ic type a is congruent to the sum of an integer of n-ic type b and an integer of n-ic type c. The symbol is extended to include any definite number of elements. New properties, together with a list of sumbols involving the n-ic types of specific integers, are derived for use in studying Waring's problem, modulo p, for a particular exponent n. Let $T_p(n)$ be the least number such that every integer is congruent to the sum of $T_p(n)$ or fewer n-ic residues. Then for primes of the form $22k + 1$, $k > 3$, $2 \leq T_p(11) \leq 4$.

An Iowa mathematics test, by Professor Orlando C. Kreider, Iowa State College.

An Iowa college mathematics test constructed for Iowa high school seniors. The results to be used by the Iowa private and state colleges for admissions, scholarships, and placement. Three members of the test construction committee were from private colleges and three from state colleges. Fifty per cent of the items test mechanics, the other fifty per cent require thinking. The test items were selected from arithmetics, algebra (I and II), geometries (plane, solid and analytic) trigonometry, and miscellaneous (statistics, sets, and logic).

An explicit example, by Professor Allen T. Craig, State University of Iowa.

An elementary explicit example of a non-normal bivariate distribution that has normal marginal distributions is given. Let $f(x,y) = f_1(x,y) +$ $k\, f_2(x,y)$ where $-$ x,y

$$f_1(x,y) = \qquad \exp\left[-(x^2 \quad y^2 - 2 \quad xy)/2(1- \quad)\right]$$

and $f_2(x,y) = f_1(x,y)\ (x^2 - \quad xy \quad y^2 - \quad)\exp \ -(x^2 \quad y^2 - 2 \quad xy)/2(1- \quad)$.

Now $f_2(x,y)/f_1(x,y) \quad M$ so that $f_1(x,y) \quad 1 \quad kf_2(x,y)/f_1(x,y) \quad 0$ if $k \quad 1/m$. Moreover, $f_2(x,y)\, dx = \quad f_2(x,y)\, dy = 0$ so that

$f(x,y)\, dy\, dx = 1$. That is, $f(x,y)$ is a non-normal joint probability density function but each marginal distribution is normal. If one wishes to take $\quad = 0$ in $f_1(s,y)$, he may at the same time replace, in the definition of $f(x,y)$, $f_2(x,y)$ by $f_1(x,y)(xy)\ \exp\ -(x^2 \quad y^2)/2$.

<u>Some integrals and series involving Legendre Associated Functions that arise in added mass theory</u>, by Professor L. Landweber, Iowa Institute of Hydraulic Research, State University of Iowa and Professor M. Macagno, Iowa Institute of Hydraulic Research, State University of Iowa, presented by Matilde Macagno.

The added mass of a sphere performing high frequency horizontal oscillations when half submerged in a liquid has been evaluated. The problem consists of solving Laplace's equation with the velocity potential vanishing on the free surface. The value of the added mass coefficient, i.e. the ratio of the kinetic energy of the fluid to the kinetic energy of the fluid mass displaced by the hemisphere, is obtained by solving finite and infinite series containing integrals of the Legendre Associated Functions. The coefficient is found to be $C_H = \frac{4}{\quad} - 1 = 0.2732\ldots$

<u>Shears and inequalities</u>, by Professor B. Vinograde, Iowa State College.

The purposes of this note are (1) to establish the non-cycling of a shear-translation procedure for solving linear inequalities, and (2) to compare the logic of a test for inconsistency with that of the elimination method as used by H.W. Kuhn, Solvability and consistency for linear equations and inequalities, American Mathematical Monthly, vol. 63, 1956, pp. 217-232.

~~a,b . Let X_1 and X_n denote the smallest and largest items, respectively, of a random sample of size n from the distribution. Let Q(x) be a measurable function essentially bounded Lebesgue on (a,b); we may assume E Q(X) = 0 and E $\overline{Q(X)}^2$ = 1. Then~~

~~Y=n f(x)dx, V=n f(x)dx, and~~

Unconditional probability, by Julius G. Baron, M. D., Iowa City, Iowa

The concept of conditional probability is used to calculate an unknown probability from known ones. This is done by a certain restriction of the possible cases. The usual expressions are "if known" or "if given." A simple example is presented in which such a formulation leads to a contradiction. This contradiction is discussed. It is shown that the expressions mentioned may involve a volitional act of an informer. This makes the use of the concept of mathematical probability impossible. If the expression "known as a result of a random experiment" is substituted, then the paradox is eliminated.

Note on a limiting distribution, by Professor Donald A. Jones, State University of Iowa, (read by title only).

Let X be a real random variable of the continuous type possessing a probability density function (pdf), f(x), which is continuous on the interval (a,b), positive almost everywhere Lebesgue on a,b and zero outside of a,b . Let X_1 and X_n denote the smallest and largest items, respectively, of a random sample of size n from the distribution. Let Q(x) be a measurable function essentially bounded Lebesgue on (a,b); we may assume E Q(X) = 0 and E $\overline{Q(X)}^2$ = 1. Then

$$Y = n \quad f(x)dx, \quad V = n \quad f(x)dx, \text{ and } \quad Z = \frac{1}{n} \quad Q(X)$$

have a limiting distribution given by the joint pdf

$$(1/\ 2\ )\ \exp\ (-y-v-z^2/2) \qquad \text{for } y\ \ 0,\ v\ \ 0, \text{ and all } z.$$

$$0 \qquad \text{elsewhere.}$$

An application of generalized means, by Professor Sidney D. Nolte, Iowa State College.

The generalized mean M (x,y) was defined to be $^{-1}$ p (x) q (y) where p, q 0, p $q = 1$ and (x) is monotone and continuous. This mean was applied to the second difference $(f; x, h) = f(x \quad h) \quad f(x - h) - f(x)$ to form a generalized second difference $(f; x, h) = M \quad f(x \quad h)$, $f(x - h) \quad - f(x)$.

A study was made of functions whose generalized second differences satisfy certain conditions. Maxima of classes of generalized quasismooth functions were examined.