

Joint Meetings of the Iowa Sections of MAA, ASA, SIAM

Science Building, Grinnell College, Grinnell, Iowa

March 26 - 27, 1982

Friday, March 26, Room 171 Science

- 1:30 Registration
- 2:00 M. H. Millar, University of Northern Iowa  
Transforms and the Consistency of Several Algebraic Theories
- 2:30 J. Murdock, Iowa State University  
The Algebraic Geometry of Elementary Integral Calculus
- 3:00 Break
- ★ 3:15 S. J. Willson, Iowa State University  
Objects of Fractional Dimension
- 3:45 W. Rudolph, Iowa State University,  
The Mathematical Teacher Shortage
- 4:14 R. Jacobsen, Luther College  
Changing Patterns in Freshman Mathematics Placement at Luther College
- 4:45 A. M. Fink, Iowa State University, MAWIS Report.

Friday, March 26, Grinnell House

8:00 - 10:00 PM Social Hour, Tickets purchased at Registration

Saturday, March 27, Room 181 Science

- 8:30 Coffee and donuts
- ★ 9:00 H. W. Hethcote, University of Iowa  
Measles and Rubella: Can They be Eradicated in the U.S.A.?
- 10:00 Marsha Sward, MAA
- 11:00 Break
- 11:15 Governor's Report
- 11:30 Business Meeting
- 12:00 Informal Luncheon for Association for Women in Mathematics members  
and others interested in AWM.
- ★ 1:30 S. S. Brier, University of Iowa, Statistics and the Law

CONCURRENT SESSIONS

Room 171

- 2:00 E. Moore, Grinnell College, Place-value Systems and Integers in the Computer
- 2:30 A. Solow, Grinnell College, Direct Proofs of Equalities
- ★ 3:00 R. K. Smith, Graceland College, Minimal Spheres
- 3:30 I. R. Hentzel and R. K. Smith, Iowa State University and Graceland College,  
Tictactoe for Strategists

Room 124

- 2:00 J. Peters, Iowa State University, Simple C<sup>\*</sup> Algebras
- 2:30 K. Atkinson, University of Iowa, Two Surface Triangulation Packages
- 3:00 E. Johnston, Iowa State University, Rearrangements of Divergent Series
- 3:30 A. M. Fink, Iowa State University, Summing Series of completely  
monotonic functions.

RONALD K. SMITH, Graceland College, Lamoni, Iowa 50140. Minimal Spheres.

A molecule in solution is thought to occupy a tiny bubble. Given the coordinates of the atoms, we want to find the center and radius of the smallest sphere that could contain the molecule. Specifically, we show how to find the smallest sphere in  $\mathbb{R}^n$  containing a given bounded set. This problem provides an interesting example of elementary analysis, and its solution leads to an interesting programming problem.

Objects of Fractional Dimension  
Stephen Willson, ISU

Abstract:

The theory of topological dimension is well-known, but the parallel theory of Hausdorff dimension is less familiar. The Hausdorff dimension is defined for all compact metric spaces and is invariant under isometries (but not arbitrary homeomorphisms). The Hausdorff dimension may be infinite or any nonnegative real number, and a subspace of Euclidean space whose Hausdorff dimension is not integral is commonly called a "fractal."

This talk will be a somewhat informal exposition of Hausdorff dimension. In particular, we shall try to see why the Cantor set is a fractal, and why Pascal's triangle mod 2 can help generate a fractal.

Changing Patterns in Freshman Mathematics Placement at Luther College

by R.S. Jacobsen  
Luther College  
Decorah, Iowa

Abstract:

This paper will discuss the need for freshman Mathematics placement at a liberal arts college, current mathematics offerings for freshman, and some procedures for suggesting courses for incoming freshman. Statistical results from the placement program at Luther College will be presented.

The Algebraic Geometry of Elementary Integral Calculus

James A. Murdock

A beginning course in complex variables can provide a chance to review and unify the treatment of integrals in an elementary calculus course, showing why certain forms are integrable and others are not. A first graduate course in complex variables can go further and point the way toward the Riemann - Roch theorem, using elementary integration as the source of motivation and enhancing the realization of the unity of mathematics.

### Two Surface Triangulation Packages

Kendall Atkinson, University of Iowa

A spherical triangulation package is described. It creates and refines triangulations on the unit sphere, and it allows a number of operations to be performed on these triangulations. By means of smooth maps on the sphere, the package can also be used to create and manipulate triangulations on other surfaces, for example, an ellipsoid. The package is written in Fortran 77, and it is portable. A second package will also be discussed, for triangulations on surfaces in  $R_3$  which are only piecewise smooth.

### Summing Series of Completely Monotonic Functions

A. M. Fink

The sums  $\sum f(n)$  or  $\sum (-1)^n f(n)$  where  $f$  is completely monotonic can be approximated very rapidly by short sums. How to find such formulae is made easier by integral representations.

Title: "Transforms and the Consistency of Several Algebraic Theories".

Michael Hillar

Abstract: The techniques employed in establishing the consistency of certain familiar formal algebraic theories (e.g., theory of groups, theory of rings, etc.) often depend quite heavily on intricate semantic constructions involving the notions of interpretation, satisfaction, and models of formal theories. Using a simple transformation performed on arbitrary formulas of certain algebraic theories, I will show how these semantic notions can be avoided and the questions of consistency settled by examining a few simple tautologies of the propositional calculus; limitations of the transform method will also be considered.

### MAWIS Project Report

A. M. Fink

The 4-video cassettes (together with a project book)

devised by MAA under a grant from NSF are to be used by secondary schools for motivation to take mathematics. The project is described.

### Simple $C^*$ -Algebras from Group Actions

Justin Peters, I.S.U.

Consider the  $C^*$ -algebra of the canonical commutation relations. Let  $\alpha$  be an irrational number in  $(0,1)$ ,  $\lambda = \sqrt{\alpha}$ , and define unitary operators  $U_\lambda, V_\lambda$  on  $L^2[0,1]$  by  $(U_\lambda g)(x) = e^{2\pi i \lambda x} g(x)$ ,  $(V_\lambda g)(x) = g(x - \lambda)$ ,  $g \in L^2[0,1]$ ,  $x \in [0,1]$ . Then  $U_\lambda, V_\lambda$  satisfy  $U_\lambda V_\lambda = e^{2\pi i \alpha} V_\lambda U_\lambda$  (the "canonical commutation relations"). Let  $\mathfrak{U}_\alpha$  be the closed subalgebra of  $B(L^2[0,1])$  generated by  $U_\lambda, V_\lambda$  and their adjoints. The algebra  $\mathfrak{U}_\alpha$  arises in other contexts as well; e.g., as a simple non-finite dimensional quotient of the  $C^*$ -algebra of the Heisenberg group over the integers. In addition,  $\mathfrak{U}_\alpha$  appears as the  $C^*$ -crossed product of the action of  $Z$  on  $C(T)$  given by  $(n \cdot f)(x) = f(x + n\alpha)$ , where the addition is understood modulo 1. For this reason  $\mathfrak{U}_\alpha$  is also called the irrational rotation algebra. We discuss crossed products from dynamical systems more generally, and show how they give rise to a large class of simple  $C^*$ -algebras.

"MEASLES AND RUBELLA: CAN THEY BE ERADICATED IN THE U.S.A.?"

By Herbert W. Hethcote  
Department of Mathematics  
University of Iowa

Abstract: An age-dependent model for the transmission dynamics of an infectious disease is presented and the endemic stationary solution for vaccination at various ages is given. Age-specific incidences are used to estimate parameter values. Even with the school immunization laws in the U.S.A., some children are not vaccinated and some do not become immune from their vaccination. Calculations show that a small percentage without immunity can prevent the eradication of measles in a population and that a second dose of measles vaccine at school entry may be necessary for eradication. The incidence of congenital rubella syndrome is compared for five rubella vaccination strategies. The U.S.A. strategy of vaccinating all children for rubella at a young age is worse than the British strategy of vaccinating only 11 to 14 year old girls when the percentage actually immunized is less than approximately 85%; however, above 85%, the U.S.A. strategy is better and can lead to eradication.

Steven S. Brier Statistics and the Law (A.S.A., University of Iowa)

Abstract:

In recent years the use of statistical evidence in support of legal arguments has grown rapidly. Prominent situations where statistical methods have been used include: cases of criminal prosecution and defence; employment discrimination; and the issue of capital punishment. We discuss the utility of appealing to statistical methods and some of their pitfalls with respect to some specific cases.

Emily Moore, Grinnell College

Abstract -- Place-value systems and  
Integers in the Computer

Computers usually store integers in two's complement notation. For positive numbers, this is identical to the binary number system -- a place-value system in which position  $i$  has value  $2^{i-1}$ . In two's complement notation, the form of negative numbers is usually described in an algorithmic fashion: Take the binary representation of the positive number, complement it (i.e., change 1's to 0's and 0's to 1's), and add 1, using the usual rules for carrying in binary arithmetic.

We first review the history of this definition, and discuss two problems -- overflow in this system and a problem given by a physicist to interpret double-precision integers -- using only the algorithmic definition. We then give an equivalent definition using a place-value system. With this second definition, we are able to give a clearer solution to the two problems mentioned.

Finally, we explore other interesting place-value number systems using material and problems from Knuth's, The Art of Computer Programming, Vol. 2.

Bill Rudolph, I.S.U., Ames, Iowa 50011. Topic: The Mathematics Teacher Shortage. Abstract: Some ideas will be presented on how to increase the number of qualified mathematics teachers.

Direct Proofs of Equalities  
Anita Solow, Grinnell

In mathematics, in particular in combinatorics, there are many examples of sets known to have the same cardinality, but where a direct bijection between the sets is unknown. For example, Young Tableaux satisfy the following recurrence relation:

Let  $t_n$  = the total number of tableaux that can be formed from  $n$  distinct elements,  
then  $t_n = t_{n-1} + (n-1)t_{n-2}$ .

The standard proof shows that both sides of the equality are equal to the same thing, namely, the number of involutions of  $\{1, 2, \dots, n\}$ . There is no known direct bijection.

In this talk, we shall mainly consider the recurrence relation that  $G_n$ , the total number of subspaces of a vector space of dimension  $n$  over  $GF(q)$ , satisfies:

$$G_{n+1} = 2G_n + (q^n - 1)G_{n-1}.$$

ELGIN H. JOHNSTON, Iowa State University, Ames, Iowa 50011. Rearrangements of divergent series.

Let  $\sum a_k$  be a divergent series of positive numbers. The rate of divergence of  $\sum a_k$  is related to the behavior of subseries and to rearrangements of the series. We show that the rate of divergence of  $\sum a_k$  can be changed, through rearrangement, to give some other predesignated rate of divergence. We also give a combinatoric characterization of those permutations that do not affect the rate of divergence of any divergent series of positive terms.

IRVIN R. HENTZEL and RONALD K. SMITH, Iowa State University, Ames, Iowa 50011. Tictactoe for Strategists.

The version of tictactoe analyzed in this paper is played on the standard tictactoe board. Instead of X's and O's, one player plays the numbers  $\{1, 3, 5, 7, 9\}$  and the other player plays the numbers  $\{2, 4, 6, 8\}$ . Since the odd player has the extra move, he plays first. The players then alternate playing their numbers on the tictactoe board. Each player is allowed to place any of his numbers in any vacant square of the board; a number cannot be played more than once. After a number is played into a vacant square, it cannot be changed. The player who places one of his numbers so that the sum of three numbers in a line (horizontally, vertically, diagonally) totals 15 is the winner. If at the end of the game no one has won, the game is a tie.

This game is far more intricate than tictactoe. It is difficult to pick the best move but not impossible. It is a new challenge for people who have mastered tictactoe and now find it too easy.