ON GROWING LARGE TREES

John Seal
Drake University

We consider the problem of augmenting a tree (acyclic graph) by allowing a single edge to be attached at any of the existing vertices. The trees constructed this way are referred to in the literature as "physical" trees and are of interest in astrophysics. Frequently distributions of such trees will be derived for various values of n, the number of vertices. In addition, we will consider alternative probabilistic models which discriminate among vertices of different degree and will discuss these probability dependent frequency distributions. Some regularities in the "shape" of the graph (the branching pattern) and the relative frequency numbers have been observed and will be discussed. Because enumeration of all physical trees for a given n has already been reported in the literature we can, at least for some "shapes", obtain the probability of the occurrence of trees of any size!

PERFECT SQUARE, TRIANGULAR NUMBERS

Philip Zee
Loras College

We define a PST number to be a non-negative integer which is a perfect square and which is triangular, i.e., one half the product of consecutive non-negative integers. For example, $36=6^2$ and $36=1/2\ (8)(9)$. A formula which generates an infinite sequence of PST numbers is sought. First, a partial sequence of ten (10) PST numbers is discovered by careful thought and good luck. From the factors of the numbers is this list, a linear homogeneous second-order difference equation is determined. This leads to a conjecture about two formulas from which infinitely many PST numbers can be easily computed.

HOW TO TIE A MATRIX TO A KNOT

John Squires
Drake University

The problem of recognizing two knots as different is at least as hard as recognizing two graphs as different, if not more difficult. Hence it is of interest to attempt to associate with various knots graphs, i.e., 0,1 matrices. In this presentation we will consider several alternative ways how to associate with a knot a graph or its 0,1 matrix. In particular we will view Gaussian codes as a source of permutations and will examine the corresponding permutation matrices. Next we will consider cubic graphs obtained from knots by "linking" the top and the bottom of a crossing place by an edge. Finally we will consider crossing points as "fusion" sites, i.e., "novel" vertices and consider the accompanying adjacency matrices.

Part of this project involves a close look at the Gauss codes for knots and we will list some of the "forbidden" permutations in an effort to have a computer-driven construction of knots on n-crossings.

LEAST-SQUARES REGRESSION SURFACE AS A PHYSICAL SYSTEM IN EQUILIBRIUM

Nuwan Nanayakkara
Iowa State University

This talk should have pedagogical interest to teachers of Statistics. Recently many interpretations have been given for the least--squares regression surface. For example, Heitmann and Ord (1985) showed that the least--squares hyperplane can be viewed as a weighted average of all possible hyperplanes that can be formed by observational set combinations. We shall show that the least--squares regression surface can be viewed as a physical system which is in equilibrium under suitably defined forces, and derive the normal equations using the principle of virtual work. These ideas will be extended to the weighted least--squares regression surface.

RESIDUAL-BASED ESTIMATORS FOR COVARIANCE FUNCTION AND VARIOGRAM

Martin O. Grondona and Noel A. Cressie
Iowa State University

Under a linear regression model with an m-dependent error process, exact and approximate (up to order n -2) expressions for the expectation of the ols residual-based estimators for the covariance function and the variogram are derived. The use of recursive-residuals based estimators is also discussed.

GOING FROM POSETS TO TOPOLOGICAL SPACES

Veronique Ziegler
Maharishi International University

A functor going from the category of posets to the category of topological spaces yields some interesting properties about the relationship between these two categories.

GOING FROM GRAPHS TO POSETS

Paul Shimura
Maharishi International University

The functor C (G) connects the category of graphs and that of posets by transforming the graph G to the poset of connected subgraphs of G. We will investigate properties of graphs which lattices when transformed by C (G).

TEACHING ABOUT FRACTALS

Stephen J. Willson
Iowa State University

Fractals already interest and motivate many students because of their intriguing computer-drawn pictures. Fractals therefore form a good vehicle for teaching about topology and geometry. The speaker will outline some of the variety of subjects where fractals play a role. He will also comment on his experience in teaching about fractals out of Michael Barnsley's new book Fractals Everywhere.

USING DISCRETE DYNAMICAL SYSTEMS TO INSPIRE CALCULUS

James T. Sandefur
Georgetown University
Visiting University of Iowa

The derivative will be used to develop stability results for discrete dynamical systems, with the mean value theorem being used to prove these results. Then, graphical techniques taught in most calculus courses (such as the use of the second derivative to study concavity) will be used to derive results for cases in which the first derivative is inconclusive. These techniques will be used in a way that requires an understanding of the connection between calculus and graphs.

Students will develop the ability to set-up and solve discrete dynamical systems that model important real world problems in areas such a population models and genetics. Combining this ability with an understanding of the derivative, they can easily learn to set-up and solve differential equations that model important continuous phenomena.

In addition, topics such as chaos, fractals, and the Mandelbrot set can be discussed with a minimal expenditure of time. By interspersing these topics throughout a calculus course, students' interest will be maintained in mathematics.

HYPONORMAL SYSTEMS OF OPERATORS: APPLICATIONS OF COMPUTERS TO INFINITE DIMENSIONAL PROBLEMS

Raul E. Curto and Paul S. Muhly
University of Iowa

An operator T on Hilbert space is called hyponormal if and only if the commutator, $[T^*,T] := T^*T - TT^*$, is positive semi-definite. Normal operators are hyponormal, but non-normal, hyponormal operators can only be found on infinite dimensional Hilbert spaces. A commuting pair of operators, T_1 and T_2 , is called a hyponormal pair provided that the 2 x 2 operator matrix, whose i,j - entry is the commutator, $[T^*_j,T_i]$, is positive semi-definite. Larger hyponormal commuting systems of operators can be defined in a similar way. We will report on our research on hyponormal systems and how it relates to some old problems about single hyponormal operators. The interest in this research for the general audience stems from the fact that computers, in particular, symbolic manipulation programs are involved in significant ways.

CHANGES IN CALCULUS

Gilbert Strang
MIT

There is a widespread feeling that calculus courses are missing some of the purpose and vitality of the subject, and that the textbooks are partly responsible. This talk will be about the development of a new textbook. The discussion will be mathematical and also personal. We show how iteration and chaos fit naturally with Newton's method, and we also discuss a fresh look at established topics--even the Fundamental Theorem. Then we hope to speak about the overall experience of writing and publishing.

A STRANGE DUAL FOR A PROBLEM ON A SQUARE

Irvin R. Hentzel and Richard Sprague
Iowa State University

We are given three distances a,b,c. The problem is to construct a square ABCD and a point P such that the three lengths PA, PB, PC are a,b,c, respectively. We also answer these related questions. For a given a,b,c, how many solutions exist? For a particular solution, when is P inside the square?

The difficulty of the construction is that the length s of the side of the square is unknown. A related problem which appears much easier is this: Given s,a,c, find a square ABCD of side length s and a point P such that PA = a and PC = c.

The problem has a surprising twist. These problems are dual. A solution to one is a solution to the other. Yet, the first construction appears intractable while the second construction is obvious.

TEACHING EXPERIMENTAL DESIGN AT A FOUR YEAR COLLEGE

Gordon Brill
Luther College

The teaching of experimental design in the environment of a liberal arts college will be discussed. The only prerequisite is an introductory statistics course. Calculus is not required. The students are primarily psychology and mathematics majors.

POWERS OF AN IDEAL IN A COMMUTATIVE RING

Daniel D. Anderson
University of Iowa

Let R be a commutative ring with identity and let I be an ideal of R. For a natural number n, the ideal I^n is defined to be the ideal generated by all products of n elements from I, i. e., $I^n = \{a_{11}a_{21}a_{31}\cdots a_{n1}+\cdots+a_{1s}a_{2s}a_{3s}\cdots a_{ns}$: each a_{ij} is in I$\}$. We consider the question of when the ideal I^n is actually generated by all nth. powers of elements of I, i. e., the question of when $I^n = \{r_1a_1^n+\cdots+r_sa_s^n$: each r_j is in R and each a_j is in I$\}$. For example, we will show that if R contains the rational numbers, then I^n is generated by nth. powers.

USING CAS AND PASCAL PROGRAMS IN TEACHING DIFFERENTIAL EQUATIONS

Douglas A. Swan
Morningside College

At Morningside College, I have started integrating both CAS (Computer Algebraic Systems) and conventional numerical programs in the teaching of Differential Equations. I will talk about the advantages of these teaching tools in promoting a greater understanding of the subject. The students are sophomores or juniors who have taken, or are taking a course in Pascal, and have finished three semesters of calculus. There are twelve in the course.

We are using muMATH (cost $150) on a single microcomputer with a hard disk drive. The differential equations package solves many differential equations of various degrees of difficulty. MuMATH is not user friendly, this is met by using predefined differential variables and a few handouts.

Interactive Pascal programs are tailored to take Euler's Method through 4th order Runge-Kutta methods. Alternate versions allow students to compare convergence rates where exact solutions are known or to solve equations where closed form integrals do not exist.

LINEAR ALGEBRA
A Smooth Transition To The Realm of Abstraction and Proof

Eric W. Hart
Maharishi International University

Linear algebra is often the transition course from more concrete computational mathematics to more abstract, proof-oriented mathematics. This transition can be difficult for students. I will discuss some teaching techniques that make the transition easier.

First of all, I will present what is called a "Unified Field Chart" for linear algebra. This chart organizes the abstract concepts of the course and relates them to the experience of the students. Secondly, I will discuss some simple, yet effective, techniques for helping students learn how to write proofs. Both the chart and the proof-teaching techniques have proven successful in a recently taught linear algebra course.

A METHOD OF IMPROVING MATHEMATICAL INTUITION IN THE CLASSROOM

Anne Dow
Maharishi International University

The expansion of mathematical knowledge requires both intuition and intellectual analysis. This is just as true in the classroom as it is in mathematics research. There has been much research into developing the ability of intellectual analysis in the classroom, but almost none on development of intuitive ability. It is generally felt that one either has intuitive ability or one doesn't. In this talk, I will describe some of the research and views of Fischbein, Skemp and others on students' intuitive understanding of mathematics, then show how the approach used in teaching mathematics at Maharishi International University develops intuitive understanding of mathematical concepts. Particular reference will be made to the intuitive understanding of infinite process, which are the basis of calculus.

ON SUMS OF POWERS OF INTEGERS

H. K. Krishnapriyan
Drake University

The formulas for the sums of the first n positive integers, their squares and their cubes are commonly found in elementary textbooks as examples of results which can be proved by induction. While less well-known, the corresponding formulas for sums of higher powers have been frequently studied and can be traced to James Bernoulli and Johann Faulhaber who observed that for odd powers, the sums are polynomials in n(n+1) once a factor 2n+1 is factored out. We will discuss a method for proving this ancient result.

A GEOMETRY COURSE FOR ART MAJORS

Catherine A. Gorini
Maharishi International University

The Department of Art at MIU has begun to require all art majors to take a course in geometry. This paper describes the course as it was first taught this year: its successes and some surprises.

RESULTS ON TERMINAL POLYNOMIALS

A. Kleiner and M. Randic'
Drake University

If G is a tree, the terminal polynomial of G is defined to be the characteristic polynomial of the matrix whose elements are the distance between the terminal nodes of G. The question of whether a terminal polynomial of order 3 represents a unique graph is related to classical problem in number theory. The relationship between the two problems will be discussed. In addition, results on the decomposition of terminal polynomials will also be presented.

GRAPHS WITH INTEGER EIGENVALUES

Milan Randic' and Bernadette Baker
Drake University

Graphs will all eigenvalues being integers are rather rare. In addition to some two dozen known cases from the literature we found a number of others. A small fraction of such graphs can be characterized and constructed, while most evade apparent characterization. Nevertheless, we observed some regularities among such graphs which will be discussed.

In addition, we examined graphs which have at least a single integer eigenvalue. Such graphs can be associated with a system of homogenous equations with integer coefficients which may be of interest in Diophantine analysis. An algorithm for identifying a homogeneous system with an integer solution will be discussed and illustrated on selected systems of homogeneous equations. The solution is a disguised graph theory problem of identifying graphs with nodal fragments associated with integer eigenvalues.