The Math Test

Following are the five problems put to 149 high school students in yesterday's Mathematical Olympiad. None of the problems has a single correct answer; they can all be solved in many different ways and the students' papers will be scored on the basis of the ingenuity of their solutions. The students had three hours to complete their answers.

1. Let a, b, and c denote three distinct integers and let P denote a polynomial having all integral coefficients. Show that it is impossible that P(a)=b, P(b)=c, and P(c)=a.

2. Prove that if a, b, and c are positive real numbers, then

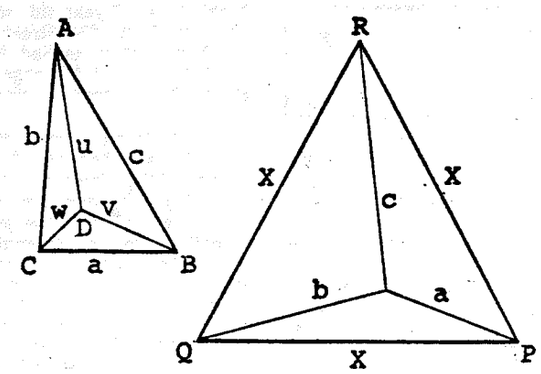
$$a^b b^c c^a \geq (abc)^{\frac{a+b+c}{3}}$$

3. Two boundary points of a sphere of radius 1 are joined by an interior arc of length less than 2. Prove that the arc must lie in some hemisphere of the given sphere.

4. A father, mother and son decide to hold a certain type of board game family tournament. The game is a two-person one with no ties. Since the father is the weakest player, he is given the choice of deciding the two players of the first game. The winner of any game is to play the person who did not play in that game and so on. The first player to win two games wins the tournament. If the son is the strongest player, it is intuitive that the father will maximize his probability of winning the tournament if he chooses to play the first game with his wife. Prove that this strategy is indeed optimal. It is assumed that any player's probability of winning an individual game from another player does not change throughout the tournament.

5. Consider the two triangles ΔABC and ΔPQR shown below. In ΔABC, $\angle ADB = \angle BDC = \angle CDA = 120°$. Prove that

$$X = u + v + w.$$



a professor of mathematics at Vanderbilt University. "When I first looked at the test I thought, 'Oh, no, I guess I'm not going to be able to prove any of them,'" he said. Then he solved the first one.

"It's kind of exhilarating when you hit upon the answer," he said. "It's interesting to see that no matter how you do it, it always come out right. Then you see the logic of it."

As a sophomore, young Arensdorf scored third highest in the country in last year's Olympiad; he skipped the 11th grade and is a senior this year.

"He already knows more than we can teach him," said his adviser, Dr. Robert Moser.

Second highest scorer in the country last year was Stuyvesant's Eric Lander, 17, who won a Westinghouse Science-Talent Search fellowship this year and was a contestant in yesterday's Olympiad.

Although the scores are not released, Eric figured that he got 75 last year and guessed that he got 83 this year.

"Wait till we get to see the papers," said Dr. Greitzer, who, along with 10 assistants, will mark all of the answer sheets and send the top 30 papers to Memphis State University, where they will be reviewed.

The winners should be announced by June 1, and a special awards ceremony will be held in Washington for the eight highest scorers on June 27.

The history of mathematics shows that it is one of the few disciplines that boasts of true prodigies. "You don't have to be intellectually mature to do mathematics," said Mrs. Henrietta Mazen, chairman of Bronx Science's math department. "You don't have to have, as you do in writing, the ability to understand other people."

Jesse Deutsch of Stuyvesant recalled: "In junior high school I learned that all the other courses are mass memorization. But math is the only subject that you have to think a bit."

At Bronx Science, James Roskind said he was going to call his friend Eric Lander last night to see how he did.

"I bet him $5 that he was going to make it into the finals," the Roskind youth said.

Wasn't he betting against himself, he was asked.

"No," he said. "I'm betting for him."

Said Jesse Deutsch of Stuyvesant: "It's nice being surrounded by guys who are the same kind of crazy you are."

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[3] THE THIRD U.S.A. MATHEMATICAL OLYMPIAD

MAY 7, 1974

RESULTS - IN ORDER OF RANK

Paul Zeitz	Stuyvesant High School	New York, N.Y.
Stephen Modzelewski	Shady Side Academy	Pittsburgh, Pa.
Gerhard Arenstorf	Peabody Demonstration School	Nashville, Tenn.
Thomas Nisonger	Walt Whitman High School	Bethesda, Md.
Eric Lander	Stuyvesant High School	New York, N.Y.
David Barton	Berkeley High School	Berkeley, Cal.
George Gilbert	Washington Lee H.S.	Arlington, Va.
Paul Herdeg	Hamilton-Wenham H.S.	Hamilton, Mass
Charles Hornig	Lexington High School	Lexington, Mass
Gilbert Chin	Lawrenceville School	Lawrenceville, N.J.
Michael Barall	Flushing High School	Flushing, N.Y.
Douglas Oman	Berkeley High School	Berkeley, Cal.
Stuart Hara	San Mateo High School	San Mateo, Cal.
Jesse Deutsch	Stuyvesant High School	New York, N.Y.
Peter Homeier	Palisades High School	Pacific Palisades, Cal.
David Lampert	Ann Arbor Huron H.S.	Ann Arbor, Mich.
Albert Zisook	New Trier Twp. H. S.	Winnetka, Ill.
Will Jagy	Syosset High School	Syosset, N. Y.
Bert Wells	Granite Hills H. S.	El Cajon, Cal.
Richard Anders	Great Neck So. Sr. H.S.	Great Neck, N.Y.
Richard Elder	Berkeley High School	Berkeley, Cal
Colin Park	Roosevelt High School	Honolulu, Hawaii
N. Christopher Phillips	Acalenes High School	Lafayette, Cal.
Gerhard Harrop	Baltimore Polytechnic Inst.	Baltimore, Md.
Steve Jackson	Oakville High School	St. Louis, Mo.
Michael Roberts } tied	Bethesda-Chevy Chase H.S.	Bethesda, Md.

The New York Times

Qualifying for World Math Olympiad Adds Up to Challenge for 149 Students

By LEE DEMBART

For three hours they pored over their papers, filling pages of scratch paper, testing hypotheses, pursuing false leads, sighing softly and gazing at the ceiling for divine inspiration, which occasionally they got.

The four students at two cluttered tables in a small room at the Bronx High School of Science were among 149 of the nation's top high school mathematicians competing yesterday for prizes, honors and a trip to Europe in the third annual U.S.A. Mathematical Olympiad.

"You just keep working and everything turns into a simple answer," said James Roskind, a 16-year-old senior from West 162d Street in the Bronx, after he had finished the test. "A full page of gibberish becomes a single line."

A fifth Bronx Science student, Richard Kenner, 16, of Forest Hills, Queens, took yesterday's five-question test in Chicago, where he is participating in a computer science fair. The five entrants gave Bronx Science more contestants than any of the other 107 competing schools. In all, 18 students at nine New York City high schools took part.

The first round of the olympiad was held in March when 350,000 students across the country took the 25th annual mathematics examination of the Mathematical Association of America. The top scorers on that test were yesterday's contestants.

The 24 highest scorers on yesterday's test will be invited to Rutgers University in June for a three-week training session to choose an eight-member team to represent the United States in the International Mathematical Olympiad to be held in Erfurt, East Germany, in July. This is the first year the United States will be competing in the 17-year-old event.

"I'm sure I got number 3 totally," said young Roskind of Bronx Science. "I just went straight through it and gave a real rigorous answer. In number 4 I just gave the method of solution, and I didn't write any answer at all for number 1."

At Stuyvesant, Jesse Deutsch, a 17-year-old senior from Canarsie, was confident that he had solved Problem 5 even though the other two contestants there had solved it differently.

"The other guys used a formula," he said. "What I did was use some geometry. I rotated the triangle. I know I got it right. They said, 'Prove it,' and I proved it!"

In Houston, Ellen Hahne, a 17-year-old senior at Memorial Senior High School, said she had found the probability problem (number 4) the easiest.

"I figured out all the possible ways that the father could win," she said, "and some of those would involve him playing the son first, some of those would involve him playing the mother first, and some of those would involve the son playing the mother first.

Yesterday's test was designed to measure not computational skills, but intuition and mathematical creativity.

'Elementary Problems'

"These are elementary mathematical problems, but they require a peculiar quirk of mind to do," said Prof. Samuel L. Greitzer of Rutgers, chairman of the olympiad committee. "We've never had anyone who got them all right. There's extra credit for ingenuity, and let me tell you, these kids show ingenuity."

The problems come from the fertile mind of Dr. Murray Klamkin, a mathematician at the Ford Motor Company in Dearborn, Mich., who is an internationally known problem solver—and creator.

In the usual post mortem that follows high school tests, the students at Bronx Science had barely finished sealing the envelopes that contained their answer sheets when they burst into a chorus of "How'd you do?"

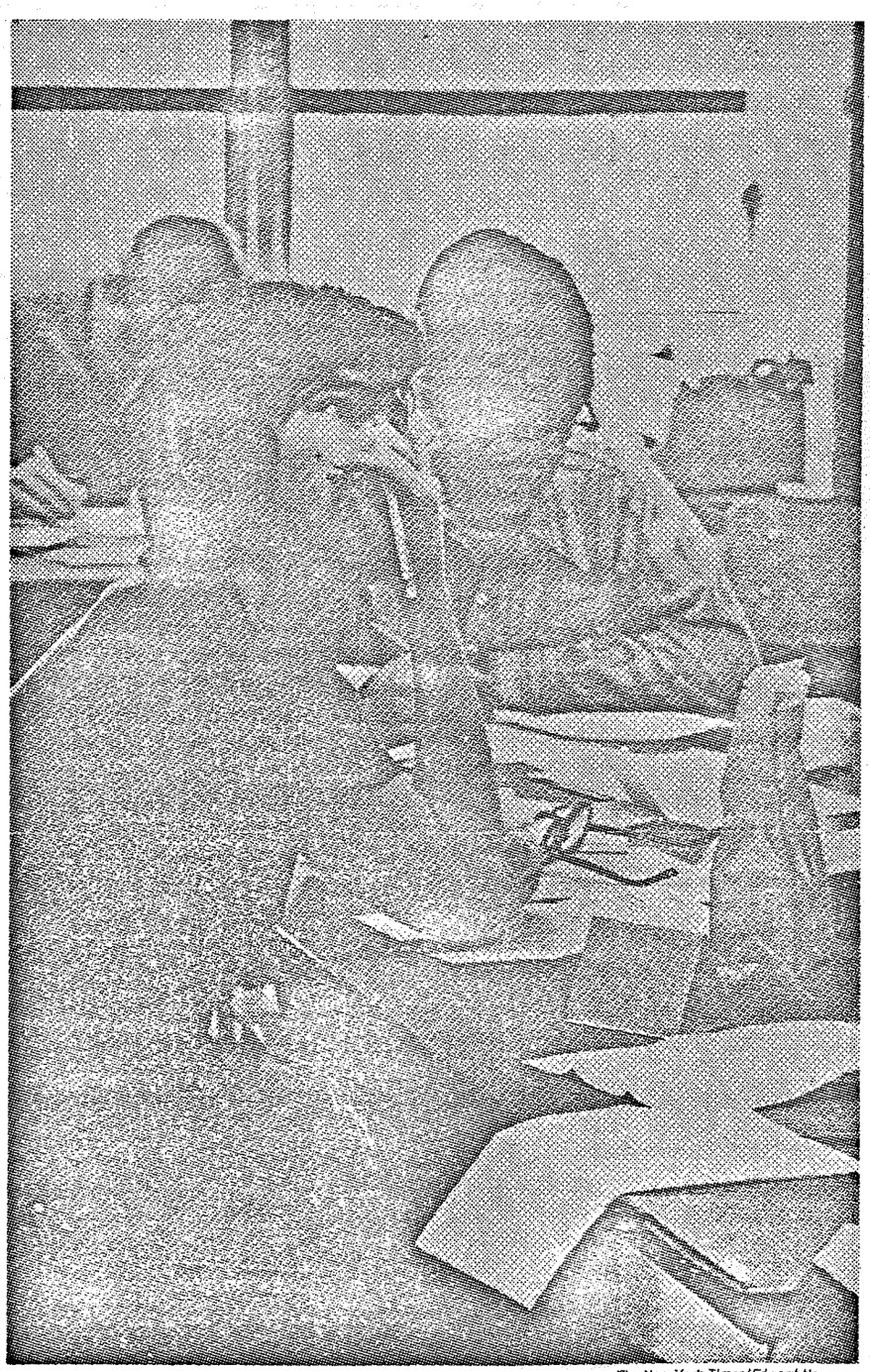
At Bronx Science it was generally agreed that Problem 1 was the hardest, but the three students who took the test at Stuyvesant High School called that one "trivial" and said Problem 3 was "the hard one."

"I divided them into three sets and added up the probability of each one."

In Problem 2, she said, she renamed b as $(a + k)$ and renamed c as $(a + i)$ "and just changed the problem so it was all in terms of a and k and l.

"It's really a simple problem," she said. "Instead of looking for extra credit for ingenuity, I just did it the old-fashioned way."

Gerhardt Arensdorf, 17, took the test at the Peabody Demonstration School in Nashville, where his father is



The New York Times/Edward Hausner

Three of the four students who took the math test yesterday in the Bronx High School of Science. In the foreground is Bernard Alpern, who had broken a finger at handball.