Requested 100 word abstract

Midwestern College

Denison, Iowa 51442

March 18, 1967

Professor Earle L. Canfield
Secretary-Treasurer, Iowa Section
Mathematical Association of America
Drake University
Des Moines, Iowa 50311

Dear Professor Canfield:

As directed by Professor Waltmann, I am sending you an abstract of the paper which I have offered to present at the April 21 meeting of the Iowa Section of the Mathematical Association of America at Drake University.

My paper is a résumé of my doctoral thesis which I wrote at the University of Minnesota in 1959 under the advisement of Professor Hugh Turrittin.

Sincerely,

Wm J. A. Culmer

William J. A. Culmer
Prof. of Math. & Physics &
Chairman of the Department

Convergent Solutions of Ordinary Linear Homogeneous Difference Equations in the Neighborhood of an Irregular Singular Point

ABSTRACT

This paper deals with the solution of the linear homogeneous difference equation

$$X(s+1) = s^h A(s) X(s),$$

where h is a constant; the independent variable s is complex; and $A(s)$ is a 2X2 square matrix representable as a power series in $1/s$, namely

$$A(s) = A_0 + \frac{A_1}{s} + \frac{A_2}{s^2} + \cdots, \qquad |s| > s_0,$$

which, as indicated, is convergent in some neighborhood of $s = \infty$. The coefficients $A_0, A_1, A_2, \cdots$ are known constant matrices with complex elements. The unknown $X(s)$ is a 2X2 matrix also.

After a preliminary transformation which removes the s^h factor, normalizing non-singular transformations are applied which reduce the constant term A_0 in the new coefficient matrix to one of the canonical forms listed below:

I. $\begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix}$; II. $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$; III. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; IV. $\begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix}$; V. $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$.

In treating the problem when a lead matrix of types II, III, IV or V arises, it turns out that a number of special situations occur and in total twelve cases and subcases need individual treatment. In all of these various cases one attempts by means of a <u>finite</u> sequence of non-singular transformations to throw the difference equation into such a form that it is satisfied by a formal power series running in powers of either $1/s$ or $1/s^{1/2}$.

In two subcases the procedure used in other cases becomes infinitely repetitive. Special transformations are used to solve these two subcases or reduce them to former cases. It is believed that these two subcases constitute the problems which G. D. Birkhoff in his 1930 Acta Mathematica paper said lead to "indefinite algebraic complications" and for which he did not give a constructive solution. In this paper, by contrast, a straight forward method of computing the formal solution is given in <u>every</u> case.

The second part of the paper considers the question of convergence. In all but the two subcases leading to series in powers of $1/S^{1/2}$, the formal solutions for large $|S|$ are shown to be asymptotic representations of true solutions in certain regions of the s-plane. The asymptotic series solutions in general diverge, nevertheless they are shown to be Borel summable and replaceable by convergent factorial series, which would permit one to compute true solutions to within any prescribed degree of accuracy.

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