MATHEMATICAL ASSOCIATION OF AMERICA, INC.

ABSTRACT OF PAPER

Title of Paper: FUNCTIONS "CLOSE" TO CONTINUOUS FUNCTIONS

Time 15 minutes.

Name of Author: ANTHONY IRUDAYANATHAN

Institution: Iowa State University, Ames, Ia.

Address: Department of Mathematics
Iowa State University, Ames: Ia.
(St. Joseph's College, Tiruchirapalli-2
Madras State : INDIA)

Member of Mathematical Association of America: Yes ☐ No ☒

ABSTRACT

The abstract should be in the form of a brief and concise statement of the main results or points of view of the paper, without demonstrations and with a minimum of formulae. It should not exceed 100 words and should be compressed if possible into a single paragraph. It should be written in the third person. The abstract should be typewritten and in a form suitable for immediate publication in the MONTHLY.

FUNCTIONS "CLOSE" TO CONTINUOUS FUNCTIONS

Let X, Y denote topological spaces, F the set of all functions on X to Y and Ω the family of all open coverings for the space Y. Definitions: (1) Let $f, g \in F$ and $\mathcal{L} \in \Omega$. Then g is said to be "$\mathcal{L}$-close" or "$\mathcal{L}$-near" to f if and only if for every $x \in X$, it is true that $f(x), g(x) \in U$ for some $U \in \mathcal{L}$. (2) A function $f \in F$ is said to be (weakly) nearly continuous if and only if for every $\mathcal{L} \in \Omega$, there exists a (nearly) continuous function $g \in F$ such that g is $\mathcal{L}$-near to f. (3) A subset $A \subset Y$ is said to be nearly connected (nearly compact) if and only if for every $\mathcal{L} \in \Omega$, there exists a connected (compact) set $B \subset Y$ such that $A \subset \mathcal{L}^*(B)$ and $B \subset \mathcal{L}^*(A)$. $[\mathcal{L}^*(A) = \bigcup\{U \in \mathcal{L} : U \cap A \neq \emptyset\}]$. The main results are: (1) Every continuous function is nearly continuous; but the converse is not true even if Y is T_0, T_1, T_2 or fully normal. However, if Y is regular (i.e., satisfies the T_3-axiom), any nearly continuous function $f: X \rightarrow Y$ is continuous. (2) The composition of a continuous and a nearly continuous function (in any order) is nearly continuous; the composition of two nearly continuous functions is weakly nearly continuous. (3) The nearly continuous image of a connected (compact) set is nearly connected (nearly compact). (4) Nearly connectedness and nearly compactness are both continuous invariants. (5) Suppose X is a set and V is a topology for X such that (X, V) is a regular space with the fixed point property. If U be a larger topology for X such that if R is open in U, its closure is the same in both topologies, then (X, U) has the nearly continuous fixed point property (i.e., every nearly continuous function of X into itself maps a point onto itself).

FUNCTIONS "CLOSE" TO CONTINUOUS FUNCTIONS

Definitions: $f \in Y^X$ is (weakly) nearly continuous iff for every open cover $\mathcal{L}$ of Y, there is a continuous $g \in Y^X$ such that for every $x \in X$, $f(x), g(x) \in U$ for some $U \in \mathcal{L}$ (i.e., g is $\mathcal{L}$-near to f). (2) $A \subset Y$ is nearly connected (nearly compact) iff for every open cover $\mathcal{L}$ of Y, there is a connected (compact) set $B \subset Y$ such that $A \subset \mathcal{L}^*(B)$ and $B \subset \mathcal{L}^*(A)$. $[\mathcal{L}^*(A) = \bigcup\{U \in \mathcal{L} : U \cap A \neq \emptyset\}]$.

Results: (1) continuity implies near continuity; converse is true if range is regular (satisfies T_3). (2) composition of continuous and nearly continuous functions is nearly continuous; composition of two nearly continuous functions is weakly nearly continuous. (3) nearly continuous images of connected (compact) sets are nearly connected (nearly compact). (4) Nearly connectedness and nearly compactness are both continuous invariants.