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The Reversed Upper Central Series in the Polarization Process in Groups

ABSTRACT

Let G be a group and $\{Z_n\}$ be the upper central series of G. If $Z_n = G$ for some positive integer n and if c is the least such integer, we define a chain, $\{K_i\}$, of subgroups of G by: (a) $K_i = Z_{c-i+1}$ if $1 \leq i \leq c$ and (b) $K_i = \{1\}$ if $i \geq c+1$. Call the chain $\{K_i\}$ the reversed upper central series of G. Now suppose we are considering a class, S, of groups defined by one or more identical relations involving commutators; for example, by the fourth Engel condition $(y,x,x,x,x) = 1$. An illustration is given on how the reversed upper central series may be used to determine a bound for the nilpotency class of those groups in S which are nilpotent and have a given number (finite) of generators and have no elements of certain prime orders. Finally, this chain is used to obtain polarized identical relations involving commutators in groups satisfying the fourth Engel condition.