ABSTRACTS OF TALKS PRESENTED TO THE INDIANA SECTION OF THE MAA

1. INTRODUCTION
The Fall 2011 meeting of the Indiana Section of the Mathematical Association of America is meeting at the University of Indianapolis, October 22. The abstracts appearing here are based on text electronically submitted by the presenters. Contributed talks are listed in alphabetical order by presenter.

2. INVITED TALK — PÓLYA LECTURE

Presenter: Jeffrey Lagarias, University of Michigan

Packing space with regular tetrahedra

The problem of determining the densest packing of space by congruent regular tetrahedra has a long history, starting with Aristotle’s assertion that regular tetrahedra fill space, and continuing through its appearance in Hilbert’s 18th problem. This talk describes its history and many recent results obtained on this problem including contributions by physicists, chemists, and materials scientists. The current record for packing density is held by my former student Elizabeth Chen, with Michael Engel and Sharon Glotzer.

3. INDIANA PROJECT NExT PANEL DISCUSSION

Panelists: Dan Callon, Franklin College; Melissa Desjarlais, Valparaiso University; David Housman, Goshen College

Writing Projects in Mathematics Courses

The panelists will discuss their perspectives and experiences regarding writing and exploratory projects in mathematics courses. The panelists will each speak, followed by a lengthy question-and-answer session. All topics regarding writing projects are fair game for discussion. (*Attendance at three Project NExT panels will allow audience members the option of joining Indiana NExT.*)
4. Contributed Talks

**Presenter:** Kurt Bryan, Rose-Hulman Institute of Technology

*Making do with less: The mathematics of compressed sensing*

Suppose a bag contains 100 marbles, each with mass 10 grams, except for one defective off-mass marble. Given an accurate electronic balance that can accommodate up to 100 marbles at a time, how would you find the defective marble with the fewest number of weighings? (You’ve probably thought about this kind of problem and know the answer.) But what if there are two bad marbles, each of unknown mass? Or three or more? An efficient scheme isn’t so easy to figure out now, is it? Is there a strategy that’s both efficient and generalizable?

The answer is “yes,” at least if the number of defective marbles is sufficiently small. Surprisingly, the procedure involves a strong dose of randomness. It’s a nice example of a new and very active topic called “compressed sensing” (CS). I’ll explain the central ideas, which require nothing more than simple matrix algebra and elementary probability, and show how researchers at Rice have used these ideas to build a “one pixel” camera!

**Presenter:** Amos Carpenter, Butler University

*The $\alpha$-recursive functions*

In this talk we give two formalisms of $\alpha$-recursive functions, one by Kripke and another by Machover. We then prove that for any admissible ordinal $\alpha$, Kripke’s $\alpha$-recursive functions are exactly Machover’s $\alpha$-recursive functions.

**Presenter:** Tyler Carrico, Indiana Wesleyan University undergraduate student

**MSC2010:** 20K01

*The probability of randomly generating finite abelian groups*

Let $G = \mathbb{Z}_{p^m} \oplus \mathbb{Z}_{p^n}$ where $m, n \in \mathbb{N}$ and $p$ is prime, and let $A$ be the event where two random elements are chosen from $G$ without repetition that together generate $G$. Following an overview of earlier results in related research, we prove the formula for the probability of $A$ occurring, and extend this result to groups of the form $\mathbb{Z}_{p^{n_1}} \oplus \cdots \oplus \mathbb{Z}_{p^{n_k}}$. Extended future research possibilities are considered.

**Presenter:** David Chapman, Valparaiso University

*On arithmetic equivalence: An algebraic approach*

If two groups $H$ and $H'$ are conjugate element by element in $G$, these groups are called locally conjugate. When these groups can be realized as Galois groups, the resulting fields are arithmetically equivalent. In this talk, we construct matrices mapping $G/H$ to $G/H'$ and use these maps to analyze the related fields.

**Presenter:** Lei Cheng, Purdue University graduate student

*Chevron structures in liquid crystal films*

In this presentation, I will begin with a brief mathematical description of liquid crystal phases. They are understood as the intermediate states between liquid and crystalline. Sharing the properties of both, liquid crystals are now widely used in display devices. Liquid crystal molecules self-organize into optimal packing arrangements where the energy is minimized. At high temperatures a liquid crystal is in the isotropic phase where the molecules’ orientations are random as in a common fluid. As the temperature is reduced, the liquid crystal enters different smectic phases
where the molecular centers of mass also align, causing the molecules to organize into layers. In particular, physicists have been interested in surface-stabilized cells where the liquid crystals are confined between close glass plates with fixed boundary conditions. Hence, the natural structure is the so-called bookshelf structure with uniform layers perpendicular to the cell plane. An interesting phenomenon occurs when the temperature is further reduced. The bookshelf structure deforms into V-shaped layers called a chevron structure. It changes the pathway of the incident light and therefore distorts the images. This structure constitutes one of the most severe obstacles towards viable LCD devices. Physicists have done a lot of experiments to try to understand this structure and control the technical difficulties resulting from it. In this talk, I would like to explore this feature analytically from a mathematical point of view.

**Presenter:** Dennis Collins, University of Puerto Rico, Mayaguez (retired)

*Measuring the symmetry of a finite group*

A method is given to measure the symmetry of any finite group according to the author’s 2011 patent and examples are given. For example, the dihedral group of the square, of order 8, has SYM symmetry = 122, and the quaternion group of order 8 has SYM symmetry = 282. The two non-isomorphic groups of order 4 have the same SYM symmetry = 7.

**Presenter:** Peter Dragnev, IPFW

**Joint work with:** J. S. Brauchart, University of South Wales, Australia; E. B. Saff, Vanderbilt University; C. E. van de Woestijne, Montanuniversität Leoben, Austria

**MSC2010:** 30C10

*A fascinating polynomial sequence arising from an electrostatics problem on the sphere*

A positive unit point charge approaching from infinity a perfectly spherical isolated conductor carrying a total charge of +1 will eventually cause a negatively charged spherical cap to appear. The determination of the smallest distance $\rho(d)$ (where $d$ is the dimension of the unit sphere) from the point charge to the sphere where still all of the sphere is positively charged is known as Gonchar’s problem. Using classical potential theory, we show that $1 + \rho(d)$ is equal to the largest positive zero of a certain sequence of monic polynomials of degree $2d - 1$ with integer coefficients which we call Gonchar polynomials. Rather surprisingly, $\rho(2)$ is the Golden ratio and $\rho(4)$ the lesser known Plastic number. But Gonchar polynomials have other interesting properties. We discuss their factorizations, investigate their zeros and present some challenging conjectures.

**Presenter:** David Finn, Rose-Hulman Institute of Technology

**Joint work with:** Todd Will, University of Wisconsin at La Crosse

*Searching for a Stokes-vergence Theorem*

In Vector Calculus in the plane, Green’s Theorem relates the circulation around a closed curve $\int_c \mathbf{F} \cdot ds$ to the curl of a vector field, the amount of infinitesimal rotation in the field at a point. Green’s Theorem also relates the flux $\int_c \mathbf{F} \cdot \mathbf{n} ds$ through a closed curve $c$ to the divergence of a vector field, the infinitesimal rate of flow out of a point. In space, Green’s Theorem for circulation becomes Stokes’ Theorem,
\[ \int_c \mathbf{F} \cdot ds = \iint_S \text{curl} (\mathbf{F}) \cdot d\mathbf{S} \] where you integrate over a surface instead of a planar region bounded by the curve \( c \). Is there a version of Stokes’ Theorem, where you integrate the flux out of a space curve and it is equal to the divergence integrated over a surface? This talk will present one possibility suitable for presentation in a multivariable calculus course.

This is joint work with Todd Will at University of Wisconsin at La Crosse, who wondered about the possibility while teaching Green’s theorem and Stokes’ Theorem at the end of a multivariate calculus course.

**Presenter:** Hui Gong, Valparaiso University

**Joint work with:** A. Thavaneswaran and D. Kalajzievska, University of Manitoba

**Closed-form of PDEs with stochastic volatility**

Recently there has been a growing interest in using stochastic volatility models in option pricing. Here, we present a unified theory to obtain closed-form expressions of conditional characteristic functions for option pricing for several stochastic volatility models, through partial differential equations.

**Presenter:** Kelly Jabbusch, Valparaiso University

**Joint work with:** Sandra Di Rocco, Royal Institute of Technology; Greg Smith, Queen’s University

**MSC2010:** 14M25

**Positivity for toric varieties**

Toric varieties are a class of algebraic varieties studied by algebraic geometers. Toric varieties come with lots of structure and have deep connections with polytopes and combinatorics. They are great testing grounds for general algebraic geometric theories, and they are concrete enough for a beginner to see many of the powerful techniques of algebraic geometry. In this talk we’ll focus on notions of positivity: we’ll introduce a classical result which connects positivity of divisors on a toric variety to polytopes, and explain recent work in progress with S. Di Rocco and G. Smith in which we connect positivity of toric vector bundles to a collection of polytopes.

**Presenter:** José Lugo, Purdue University graduate student

**MSC2010:** 46Lxx

**Quasi-diagonality of continuous fields of \( C^* \)-algebras**

Quasi-diagonal \( C^* \)-algebras form a large class of \( C^* \)-algebras and arise naturally in many contexts. Dan Voiculescu has shown that quasi-diagonality is a homotopy invariant, and consequently, that the cone of a \( C^* \)-algebra is always quasi-diagonal. In this talk we discuss an extension of this result for exact continuous fields of \( C^* \)-algebras. As an application, we obtain that the group \( C^* \)-algebras of certain central group extensions are always quasi-diagonal.
**Presenter:** Rodney Lynch, IUPUI graduate student

*An elliptic curve with complex multiplication by the square root of $-2$*

I’ll first give a quick overview of the group law on an elliptic curve and the notion of complex multiplication. Then I’ll determine an elliptic curve with multiplication by $\sqrt{-2}$ and a formula for $\sqrt{-2} \cdot (x, y)$ where $(x, y)$ is a point on the curve. This gives an endomorphism $f$ of the curve which satisfies $f(f(x, y)) = -2(x, y)$.

**Presenter:** Jeff Oaks, University of Indianapolis

*At the intersection of mathematics, history, and philosophy: Jacob Klein’s interpretation of the algebra of François Vieta*

The philosopher Jacob Klein (1899–1978), student of both Husserl and Heidegger, identified the creation of symbolic mathematics in the works of the French mathematician François Vieta (d. 1603). Klein framed his analysis of Vieta’s algebra in phenomenological terms in two long articles published in German in 1934 and 1936, and later translated into English as the book *Greek Mathematical Thought and the Origin of Algebra*. The book, written in a strictly scholarly style, is difficult for modern English readers. I will present the main idea of the second part of the book, regarding the second-order intentionality of Vieta’s “species”.

**Presenter:** Horia Petrache, IUPUI Department of Physics

**MSC2010:** 26A09

*Abstract trigonometry helps abstract thinking*

Recently, I have asked students in my undergraduate and graduate physics classes to define the sine function and to explain why the derivative of sine is the cosine. The initial puzzled reaction was followed by intense discussions around the properties of these well known functions which then led to the question of what properties of trigonometric functions can be derived from others without prior knowledge. I will discuss an approach to this question in which trigonometric functions are handled with no explicit functional (algebraic) form to determine what properties are fundamental (from which all other properties can be derived). This approach can help students enhance their ability to think abstractly when studying functions in general in addition to acquiring more insight into trigonometric functions.

**Presenter:** Morteza Seddighin, Indiana University East

*The application of the two nonzero component lemma in statistics and econometrics*

In the study of Kantorovich type inequalities and their generalizations, the presenter has discovered a proved a dimension reduction lemma which has applications in many other areas of mathematics. In this talk we will focus on the applications of this lemma in statistics and econometrics. This includes topics such as homologous correlation, efficiency of least squares, and resource allocation problems. The two nonzero component lemma states that the maximizing vectors for certain functionals have at most two nonzero components.
Presenter: James Valles, Jr., Saint Mary-of-the-Woods College

Logarithmic potential on a square
The placement of potential charges on conductors is a problem that dates back to J. J. Thomson and his study of the ground state of spherical shells of electrons. When considering the energy between two charges is described by a defined norm for those charges, the problem statement boils down to finding the possible sets of placements of said charges so that the energy of the system is as small as possible.

This talk will examine the extremal placement of three logarithmic charges located on a square which is then deformed into a rectangle and finally deformed into a line segment. The placement of these charges will be discussed as will the “jump” that occurs in order to maintain a minimum energy for the system of charges.

Presenter: Jeffrey Watt, IUPUI

Joint work with: Katleen Marrs, Andrew Gavrin, Charles Feldhaus, and Stephen Hundley

MSC2010: 97D

Central Indiana STEM Talent Expansion Program: First year results
The results of the first year’s activities of CI-STEP will be discussed. The CI-STEP is an NSF funded project to create a central Indiana pipeline to increase the number of students from the greater Indianapolis region obtaining STEM degrees. The goals of this project are to increase the numbers of students of all demographic groups who: (1) pursue STEM academic and career pathways; (2) participate in STEM research, industry internships, and honors activities; (3) graduate with an undergraduate degree in STEM fields; and (4) transition into industry, graduate and professional programs. The program has set a target of increasing the number of STEM graduates at IUPUI by 10% per year (an increase of an additional 782 STEM graduates by 2015).

The intervention strategies include: Project Seed (high school internships), Summer STEM Bridge Academies, peer-mentoring and academic advising support for transfer students, Peer-led Teaching and Learning and Just-in-Time Teaching, Honor seminars, and Career Development services and internships. By blending research on teaching and learning, with the participation in teaching and training by students from diverse backgrounds, including underrepresented minorities, it is anticipated that a significant increase in both knowledge of “what works” in successful college programs, as well as the successful result of these interventions — increasing numbers of talented graduates, will occur.

Presenter: Young Hwan You, Indiana University East

The necessary geometric condition for the solvability of the inhomogeneous Cauchy-Riemann equation
One of the ways to find an analytic function is to solve the inhomogeneous Cauchy-Riemann equation. In this talk, we introduce the basic notions and discuss the necessary conditions to solve the inhomogeneous Cauchy-Riemann equation in several complex variables.