

## ABSTRACTS OF TALKS PRESENTED TO THE INDIANA SECTION OF THE MAA

### 1. INTRODUCTION

The Fall 2009 meeting of the Indiana Section of the Mathematical Association of America was held at Purdue University Calumet, October 17. The abstracts appearing here are based on text electronically submitted by the presenters. Contributed talks are listed in roughly the order in which the presenters submitted their abstracts.

### 2. INVITED TALKS

**Presenter:** Mike Bolt, Calvin College

*Paint by number: a visualization of complex functions*

One challenge to understanding complex analysis is the difficulty one can have in forming an intuition for analytic functions. Frank Farris found a new way to visualize complex functions. The idea is to associate numbers with colors and to paint a domain with the values of the associated function. In this talk we describe different implementations of domain coloring and contrast it with the usual transformational approach. We also use domain coloring to illustrate some of the nice theorems in complex analysis. This includes some recent work with undergraduates that combines the transformational and domain coloring methods.

**Presenter:** Nicoleta Tarfulea, Purdue University Calumet

*A computational model for tumor-angiogenesis and intervention strategies for cancer*

In recent years, tumor-induced angiogenesis has become an important field of research because it represents a crucial step in the development of malignant tumors. The process is regulated by the interactions between various cell types such as endothelial cells and macrophages, and by biochemical factors. These include angiogenic promoters (e.g., vascular endothelial growth factor — VEGF) and inhibitors (angiostatin). A better understanding of its steps may contribute to the development of new cancer therapeutic strategies.

We present a hybrid mathematical model in which cells are treated as discrete units in a continuum field of a chemoattractant that evolves according to a system of reaction-diffusion equations, whereas the discrete cells serve as sources/sinks in this continuum field. It incorporates a realistic model for signal transduction and VEGF production and release, and gives insights into the aggregation patterns and the factors that influence stream formation. In particular, it serves as a tool for investigating tumor-vessel signaling and the role of mechano-chemical interactions of the cells with the substratum.

## 3. CONTRIBUTED TALKS

**Presenter:** Young Lee, Manchester College

*A modeling perspective of DSH*

Deliberate self-harm (DSH) or self-injury (SI) is the act of deliberately harming your own body, such as cutting or burning yourself, without suicidal intent. It has especially become a problem among adolescents and college-age students in institutional settings such as boarding schools, college dorms, Greek houses, and community treatment centers. I created a deterministic epidemiological model of DSH and studied the impact of actual peer pressure, virtual peer pressure (the Internet) and treatment. The discussion was done analytically, through stability analysis and sensitivity analysis, and numerically.

**Presenter:** John Lorch, Ball State University

**MSC2010:** 05B15

*A quick construction of mutually orthogonal sudoku solutions*

We introduce a transversal method for generating orthogonal mates of sudoku solutions. We show that the method produces an orthogonal mate for every linear sudoku solution of parallel type, and we characterize the linear sudoku solutions of non-parallel type for which the method works.

**Presenter:** Daniel Maxin, Valparaiso University

*Asymptotically autonomous systems*

In mathematical modeling, the situation is often encountered where a dynamical system is not autonomous, i.e., the coefficients of the state variables depend explicitly on the time. Sometimes, these coefficients converge to a finite limit, leading to a limiting Autonomous System. It is often tempting to analyze the simpler limiting system and to extrapolate stability results to the original non-autonomous system. However, this is not always possible unless certain special conditions are met. An example will be presented that illustrates how the non-autonomous system and its limiting system may have different behaviors. We will also discuss several mathematical biology models where the non-autonomous systems may arise.

**Presenter:** William J. Obermeyer, Purdue University Calumet, graduate with B.S. in Computer Science

**Faculty Mentor:** John J. Coffey

*Population process model: A computational analysis derived from Markov computations*

When medication for a disease does not provide an indication of one hundred percent remission, a mathematical simulation can provide treatment strategies through the evaluation of a stochastic model analyzed with the computational power of modern microprocessors. The behavior of the AIDS virus provides such an opportunity. The stationary distribution indicated by a state space representing the model could provide information regarding indication of the most effective choice of treatment based on a given set of variable parameters, which affect a simulation of the T-cell life cycle during AIDS: healthy T-cell production, infections that occur from infected cells, infections that occur from free virus, infected cells killed by the immune system, infected cells killed by the virus, and apoptosis, the natural death of healthy T-cells.

The goal of this study, based on a mathematical model to evaluate the simulation of the behavior of viral infection at a cellular level proposed in *A Population-Process Model for HIV Infection*, by Dr. John J. Coffey, is to identify valid sets of parameters associated with the T-cell life cycle in the presence of the AIDS virus. This paper discusses the utilization of the application of numerical computation and analysis and distributed computing applications to determine solutions and evaluate patterns that support Dr. Coffey's proposed model.

The noteworthy goal of this project will support the work of Dr. Coffey to provide more effective information regarding treatment applications, which might lead to higher success rate in the control of AIDS and HIV infection.

**Presenter:** Eric Nelson, Purdue University Calumet undergraduate student

**Joint work with:** Allison Blink and David Turpin, Jr.

**Faculty Mentor:** Nicoleta Tarfulea

*A CTL-inclusive mathematical model for antiretroviral treatment of HIV infection*

Treatment of HIV infection has traditionally consisted of antiretroviral therapy (ART), a regimen of pharmaceutical treatments that often produces unwanted physical side effects and can become costly over long periods of time. Motivated by a way to control the spread of HIV in the body without the need for large quantities of medicine, researchers have explored treatment methods which rely on stimulating an individual's immune response, such as the cytotoxic lymphocyte (CTL) response, in addition to the usage of antiretroviral drugs. This paper investigates theoretically and numerically the effect of immune effectors in modeling HIV pathogenesis; our results suggest the significant impact of the immune response on the control of the virus during primary infection. Qualitative aspects (including positivity, stability, uncertainty, and sensitivity analysis) are addressed. Additionally, by introducing drug therapy, we numerically analyze the model to assess the effect of treatment. Our results show that the inclusion of the CTL compartment produces a higher rebound for an individual's healthy helper T-cell compartment than does drug therapy alone. Furthermore, we quantitatively characterize successful drugs or drug combination scenarios.

**Presenter:** Gokarna Aryal, Purdue University Calumet

*On the appropriateness of transmuted Weibull distribution for modeling warp breakage data*

The Weibull distribution has been extensively applied to data analysis arising from several different areas. Not only the common 2-parameter and 3-parameter Weibull distributions, but also various other models developed based on Weibull distributions, are applied. In this talk we present a new generalization of the Weibull distribution which can be used to model the reliability data. We present the usefulness of the subject model to study the warp breakage data during weaving. A comparison with other generalizations of the Weibull distribution is also presented.

**Presenter:** Andrei Tarfulea, University of Chicago undergraduate student

**Joint work with:** Andrey V. Grinshpun, Carnegie Mellon University, and Pakawat Phalitnonkiat, Cornell University

**Faculty Mentor:** Sasha Rubin, Cornell University

*Parity games redefined through alternating nested traps*

Parity Games involve two players, EVEN and ODD, moving a token on a finite graph, in competition for the largest prioritized vertex occurring infinitely often in the play. Parity games gained major attention from the Model Checking problem in Modal  $\mu$ -Calculus and are of particular significance in Computational Complexity; they remain one of the few natural problems known to be in  $\text{NP} \cap \text{co-NP}$  but not known to be in P. To every parity game we associate a natural number called its *Trap-Depth*. Via purely structural means, the trap-depth and its construction embody the techniques each player may use to restrict the other's options. This number, bounded by the size of the graph, characterizes games on which a given player has non-empty winning region. Moreover, we supply an algorithm that runs in polynomial time which solves parity games of trap-depth 1. Future work may extend this to a polynomial time algorithm for parity games of arbitrary trap-depth.

**Presenter:** Amos Carpenter, Butler University

*The  $\alpha$ -valid formulae*

In this talk we define a language  $L_\alpha$  for any countable admissible ordinal  $\alpha$  and prove the  $\alpha$ -completeness theorem, i.e., a formula of  $L_\alpha$  is provable in  $L_\alpha$  if and only if it is  $\alpha$ -valid. This is a generalization of the completeness theorem of Gödel for first-order theories, i.e., a formula of a theory  $T$  is provable (a theorem) in  $T$  if and only if it is valid in  $T$ .

**Presenter:** Michael Karls, Ball State University

*The Mathematics of Star Trek*

We look at the challenges of designing a physics-based mathematics course for a general audience composed of both math and non-math majors. This course, "The Mathematics of *Star Trek*," is designed "from scratch" and based on popular culture and historical topics. We will outline the topics chosen and give examples of homework and discussion questions used.

**Presenter:** Adam Coffman, Indiana - Purdue Fort Wayne

**Joint work with:** Yifei Pan, Nanchang University and IPFW

**MSC2010:** 35B05, 35R45

*Differential equations with no small solutions*

The differential equation  $f'(x) = (f(x))^{2/3}$  has the well-known property that its solutions are not uniquely determined by an initial condition. I will consider this from another point of view, showing that a non-zero initial value at the center of an interval implies large values near an endpoint. This phenomenon can be generalized to several variables, higher order equations, and differential inequalities.

**Presenter:** Patrick Frey, IUPUI and Purdue University

**MSC2010:** 97D50

*What does linear term divisibility have to do with factoring quadratics?*

Why must one of the terms in the binomial factors of  $112x^2 - 125x - 50$  be a multiple of 16? And why can't a constant of 25 appear in either binomial? Neither of these questions is particularly hard to answer, yet such considerations are seldom used when factoring. A proposed method of factoring takes into account the connection between the prime factors of the linear term and those which will reproduce the nonlinear terms. The method greatly simplifies the factoring of the given trinomial, and works as well as or better than trial and error or the AC method for any trinomial with similar or smaller coefficients. Personal experience suggests that the proposed method might empower new learners to quickly master factoring, while also becoming more proficient at mental math.

**Presenter:** David Housman, Goshen College

**Joint work with:** Seth Unruh, undergraduate student

*An extremely simple fair division problem*

A uniform chocolate bar is to be divided among  $n$  people with perfectly accurate measuring and cutting devices. It would therefore be trivial to give each person an equal share of the chocolate. The only complicating feature of this problem is that each person places a different monetary value on the chocolate and all are willing to exchange money as well as receive portions of the chocolate. We characterize the extreme points of the convex set of envy-free divisions.

**Presenter:** S. Allen Broughton, Rose-Hulman Institute of Technology

*Roll-ups and differential geometry*

We all know that cylinders and (frustums of) right cones can be formed by rolling up a flat strip of paper, metal, plastic, or other flexible material. In fact there are pictures of such "roll-ups" in Calculus books. However, what happens when we do not have such a standard cone shape? What region do we cut out of the paper or metal to achieve a desired cone shape? The problem started as a phone call from a local manufacturing design company who had to solve this problem. They wanted to build a specific shape but did not know what the flattened out shape would be. Since their plan was to build the part from a flattened sheet of metal, the answer to the roll-up problem was crucial. In this talk we discuss the geometry problem and show a solution using the techniques of differential geometry. The techniques are not advanced, in fact everything can be done with multi-variable Calculus and the simple separation of variables in Differential Equations. The time for the talk does not allow for a complete discussion of the "ghastly derivations" but we will discuss the formulas that allow us to solve the practical problem. The formulas can be evaluated using numerical integration (Calculus II) and we show the flattened out shape from the given problem.