2017 ISMAA UNDERGRADUATE STUDENT ABSTRACTS

Devin Akman, University of Illinois at Urbana-Champaign

Music Playlists and the Digamma Function

Suppose you have a playlist of N songs and a music player which plays them at random with replacement. The first repeated song must occur between the second and (N+1)th plays. However, you'd like to know around when you can expect it to happen. We will derive the probability distribution for the time of the first repeat and use it to answer this and related questions. The methods of analysis are well-suited for treating this seemingly discrete problem.

The talk is appropriate for undergraduates, graduates, post-doctoral researchers, junior faculty, senior faculty, gamblers, music lovers, vegetarians, Uber drivers, and federal employees.

Sadia Ansari, Loyola University Chicago
Other authors: Dr. Aaron Lauve

Minimal Generating Sets of the Symmetric Group

The goal of this project is to analyze the minimal generating sets of the symmetric group S_n. To accomplish the task, we first build all of them for S_3 through S_5. We use group automorphisms and cycle-type to facilitate this. Specifically, we organize our search for minimal generating sets by the cycle-types of its elements, and we identify any such X with any of its images under conjugation. As such, “orbit size” becomes the first interesting aspect of the project. Given a minimal generating set X from an orbit, we construct the rooted tree such that each node is an element w of S_n. Its path to the root represents a shortest expression for w in terms of the generators. The properties (such as depth and width) of such trees, uniqueness up to automorphism, the posets of minimal generating sets not of the form {(1,2), (2,3), … , (n-1, n)}, and the minimal generating sets (for n = 3,4,5) that fit into a family for any n ≥ 3 are studied. (Preliminary report of work started under the auspices of the McNair program at Loyola Chicago.)

The talk is appropriate for undergraduates, especially those who have taken a first course in group theory.

Reinhardt Awender, Southern Illinois University Carbondale

Integral Apollonian Circle Packings

We will review the Euclidean geometry of Apollonian circle packings starting with Apollonius’ Theorem and Descartes’ Theorem. In addition, we will show how to construct a packing that produces integer values for the curvature of the generated circles. We will then discuss ways to count the number of circles in a packing, allowing us to touch upon some Diophantine properties of the circles. Finally, we will consider some of the ways that prime numbers appear in such packings.

This talk will be appropriate for undergraduates.

Chand Bhanot, Benedictine University

Monopoly: The "Speed Die"

Monopoly is a classic board game, played for generations by millions of people. However, in 2007, Hasbro, the current manufacturer of Monopoly game sets, introduced a new optional rule for players to use: the "Speed Die". This feature was introduced to address players' chief complaint about the game: it takes too long to play! The "Speed Die" gives players a third die to roll, potentially allowing players to purchase a second property during their turn or to have more control over where their piece moves to. Our research focused on two key questions:

1. Does playing with the "Speed Die" change which properties are more valuable than others compared to playing with the standard rules?
2. Does playing with the "Speed Die" favor a more aggressive player vs. a more conservative player, or does a player's strategy really not affect the overall outcome?

This talk will present all results we have gathered so far as well as our analysis and answers to the two key questions.

This talk is very appropriate for undergraduates, as it is being presented by an undergraduate, and concerns research conducted under the supervision of an undergraduate professor.

Chris Bruno, University of St. Francis

Affect of DFTD on Tasmanian Devil social structure

The Tasmanian Devil population has been drastically affected since the observed emergence of Devil Facial Tumor Disease in 1996, with local extinctions observed in various regions of the island of Tasmania. The devil population has seen a major change to its social structure in areas where the disease is present, with older devils becoming all but non-existent. This depletion of breeding age devils has caused the juvenile devils to reproduce much more frequently where the disease is present in the population. In this talk, a system of ODEs is presented to explore the way this change in age structure is affecting the ability of the Tasmanian Devil population to survive this disease.

This is by an undergraduate, most ideally for other undergraduates.

Julia Buczek, Roosevelt University
Other authors: Jacob Rubinstein, Luis Navarrete
Autonomous cars and their effect on traffic flow

In doing the Mathematical Competition in Modeling for 2017, we chose to model the effects on traffic flow when self-driving cars are introduced to highways. Our model introduced the conceptual ideas behind the behavior of self-driving cars, and used mathematics to support our hypothesis. We are heading towards a technology filled future and autonomous cars are going to be the norm, so we wanted to create a model in order to begin understanding the relationship between self-driving and manually driven vehicles.

Thomas Campbell, Southern Illinois University Carbondale
Improving Numerical Integration Estimates with Inverse Functions

Estimating the integral for power functions with powers $0<p<1$ takes much more computational power via standard numerical methods than functions with powers $p>1$. This is, at least partially due to the fact that such power functions are not Lipschitz continuous. However, as was first shown by C.A. Laisant in 1905, the integrals of certain functions can be expressed as integrals of their inverses. This paper hopes to show that by expressing hard to approximate integrals as integrals of their inverses, methods such as Newton-Cotes formulas and Monte Carlo methods become much more useful and effective than by approximating the integral of the original function alone.

This talk should be appropriate for students with a basic understanding of probability theory, and Calculus.

Kassy Candelaria, Benedictine University
Viabilities of Annuals vs. Perennials Over Multiple Seasons

Plants that reproduce once (annuals) and plants that reproduce more than once within their lifetimes (perennials) both exist in nature. The question is whether there are certain environments or cases in which annuals will be favored over perennials and what factors dictate this. For this work, numerous computer simulations were run based on a model taking both internal (e.g., growth rate) and external (environmental effects) effects into account on plant populations over multiple seasons. We will be discussing what was done with these simulations and our results, including in which cases annuals are favored over perennials (and vice versa).

This talk is appropriate for undergraduates, as this talk was done as part of a research project by undergraduates.

Stefano Chiaradonna, Benedictine University
The Dynamics of an Epidemiological Model for Human Papillomavirus with Partial Vaccination in a Heterogeneous Population

Human papillomavirus (HPV) is the most prevalent sexually transmitted disease in the United States. HPV strains-16 and-18 are the primary agents of cervical cancer; HPV-6 and HPV-11 are responsible for most genital warts and juvenile-onset recurrent respiratory papillomatoses. Highly efficacious vaccines have been developed to prevent these high-risk types of HPV, which are typically administered in three doses except for younger adolescents. We propose and analyze a mathematical model that investigates the implications of having a portion of the population not completing the vaccine regimen. Our model also considers the impact of having portions of the population not receiving the full-vaccine regimen.

Margaret Fortman, Lake Forest College
Image Processing and Restoration under Atmospheric Turbulence

When an image is taken while subjected to a temperature gradient, the image undergoes geometric distortion due to turbulence. One can develop and run an algorithm against a turbulent image sequence in order to remove distortion. Over an 8-week program this summer, my research team for the San Diego State University REU Program worked on developing algorithms to address such problems. I will present our exploration of various approaches and techniques of image processing, such as lucky region and super-resolution.

Zeid Ghalyoun, Southern Illinois University
A Solution to Pell's equation

A classical problem in the history of mathematics was calculating all the integral solutions (m, n) for a fixed $D > 0$ of $m^2 - D \cdot n^2 = \pm 1$. In fact, as long as D is not a perfect square, there are infinitely many distinct pairs (m, n) satisfying the solution. One reason the solutions were sought in the past was due to the good approximations the solutions gave; $\frac{m}{n} \approx \sqrt{D}$.

This talk aims to provide a solution to Pell's Equation by introducing some elementary tools from Algebra and Number Theory, and giving similar solutions to other special higher degree cases like that of Pell's equations.

Matt Gilsdorf, Benedictine University
Other authors: Katharine Howard
Using the Shingles Vaccine to Stop the Spread of Chickenpox

Shingles is a very painful infection caused by the varicella zoster virus (VZV), which is the same virus that causes chickenpox. Because shingles only occurs after someone has been infected with chickenpox and requires the person's immune system to be weakened in some way, shingles tends to occur mainly in adults over the age of 50. However, when an adult has an active shingles infection, he/she is able to spread the virus, and hence chickenpox, to those who have no immunity. Currently, the shingles vaccine Zostavax can help prevent shingles from occurring; however, it is only indicated for adults 50 and older. Because someone with an active shingles infection can spread chickenpox, and shingles can occur at any age, our goal was to try and answer the question: Could using Zostavax for all people who have had a chickenpox infection not only help control shingles, but also have a significant impact on the spread of chickenpox? This talk concerns the setup and discussion of a mathematical model of the spread of chickenpox and shingles along with the implementation of a vaccine to compare the spread of chickenpox under different availabilities of the shingles vaccine, including if a new proposed shingles vaccine, Shingrix, becomes available.

This talk is appropriate for undergraduates, as it is based on research conducted by undergraduates.

Kyle Keen, Benedictine University
When will I ever use this?

In this talk I will explore a few difficult topics covered in high school algebra. We will discuss strategies for better understanding these topics, and look at practical applications to Cryptography.

Definitely appropriate for undergraduates, especially those going into High School teaching.

Rachel Majerczyk, Benedictine Unviersity
Extinction and Permanence in a Integrated Pest Management with a Holling Type II Predation Rate

We investigate an integrated pest management (IPM) model using impulsive differential equations, with a Holling Type II predation rate using a class of impulsive differential equations. The model features a two-stage structure for the predator and prey class species, a birth pulse for the prey species, and a Holling Type II predation rate. The impulsive events of birth pulses, application of pesticide, and predator augmentation occur periodically. We establish conditions for which the model exhibits a globally aysmptotically stable total pest eradication solution and conditions for which the system is permanent.

This talk is appropriate for undergraduates.

Brianna Martin, Southern Illinois University of Carbondale
Natural Computations in Gene Regulatory Networks

The purpose of this research is to implement computability theory in gene regulatory networks. A gene regulatory network is a collection of molecular regulators that interact with each other and other elements in the cell to output gene expression levels. These networks exhibit natural computation. The class of partial recursive functions is the smallest class satisfying the following five axioms: the successor function, the constant function, projection functions, composition, and recursion. This talk describes partial progress to realizing these axioms in gene regulatory networks.

Anthony Mendoza, Benedictine University
Extinction in a Two prey One predator model for Integrated Pest Management

We investigate a two-prey, one predator model for integrated pest management (IPM). The model features a two-stage structure for each species and birth pulses for the prey species. The impulsive events of birth pulses, application of pesticide, and predator augmentation occur periodically. We establish conditions for which the model exhibits a globally asymptotically stable total pest eradication solution.

Elizabeth Rodriguez, Benedictine University
Agent-Based Model for Integrated Pest Management with Periodic Control Strategies

We consider an agent-based model (ABM) for integrated pest management (IPM). The model incorporates stage structure for both the pest species and the predator species. In this model, the two control strategies of augmentation of predator species and application of pesticide and the pest births occur periodically at possibly different frequencies. We determine conditions under which either the pest species is eradicated or both species persist. We also investigate how varying the frequency of the augmentation of the predator species and the application of pesticide with respect to the frequency of the pest births affects the amounts of augmentation and pesticide needed to obtain pest eradication and permanent solutions. We then compare pest eradication and permanent solutions in the ABM to those in an analogous model using impulsive differential equations.

Marko Saric, Benedictine University
Fractals And Iterated Function Systems
Fractals are self-similar structures which can be defined by an iterative process. In this paper, we use a computer program and iterated function systems (IFSs) to study the role of matrix norms and its effect on the type of fractal generated. In particular, we propose a connection between the spectral norm of the matrices in the IFS and the type of attractor that is graphed.

Linda Shaw, Benedictine University
A Linear Approach to Principal Component Analysis
Principal Component Analysis (PCA) is a statistical technique with a strong foundation in linear algebra. The purpose is to narrow down a wide selection of random variables to simplify the visualization of the data, discover which variables are correlated, and find which variables are the most significant in describing the data set. By expanding fundamentals of linear algebra along with applying basic knowledge of statistics, the foundations of Principal Component Analysis can be constructed.

William Stowe, Augustana College
A Recurrence Formula for Spreads
A spread is a set of points such that no three are collinear. We will prove that every spread has a convex hull that surrounds all other points. If we take the points in the spread, but not in the convex hull, this set of points is also a spread, and thus has a convex hull. The list of the orders of these convex hulls make up the genus of the spread. We will prove that there is a recurrence formula for the number of genuses a spread of order n can take. We will show that the p-th term divided by the (p-1)-th term approaches a constant, and that this constant can be written as a continued fraction.
This talk is appropriate for undergraduates.

Logan Timmons, University of St. Francis
Other authors: Renee Martin
A Markov Chain Analysis of NFL Overtime Rules
In this paper, we consider the National Football League's rules for overtime. We use Markov chain models to represent sudden death, modified sudden death, and our theoretical alternative modified sudden death. Through our model analysis, we find the average length of overtime and the probability of the team possessing the ball first during overtime winning the game. Our analysis shows that the modified sudden death rule change increased the average overtime from 7 minutes 1 second to 7 minutes 37 seconds and that the probability of the team possessing the ball first winning decreased from 59.9% to 55.4%. We propose a system where both teams are required to possess the ball at least once before the game ends and conclude this system would increase overtime length to 7 minutes 56 seconds and the probability of the team possessing the ball first winning would be 54.2%.

Dat Tran, Augustana College
Other authors: Lan Dang
The Skyrim Problem
In the game of Skyrim, players collect herbs that can be mixed together to form potions. There are 93 different herbs, each herb has 4 traits from a list of 52 total traits. The traits are initially hidden from the player. The player may mix 2, or 3 herbs together, to try and form a potion. A potion is created if two or more herbs share a common trait(s). All traits that herbs have in common are revealed on the corresponding herbs. The question becomes: How many mixes will it take to reveal all the traits on all of the herbs? To answer the question, we turn this problem into an integer linear program, and prove that it is NP-hard. We discuss and implement a general greedy probabilistic algorithm to solve the problem for the list of 93 herbs in the original game of Skyrim.

Kyle Zeberlein, Augustana College
Modeling Building with Fantasy Football Quarterbacks
We will present a system for predicting fantasy points scored by quarterbacks who have started 48 or more games in the NFL based on four independent variables. These variables are the quarterback's previous season, the quarterback's career performance against a given team, the quarterback's career performance excluding the previous season, and the defense's average fantasy points allowed to quarterbacks in the previous season. We will then use model building techniques to create a new model that will more accurately predict the quarterback's fantasy performance for a given game.
This talk is appropriate for undergraduates.