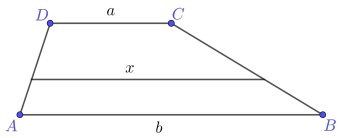
Student Mathematics Competition
Illinois Section of the
Mathematical Association of America
March 13, 2020

These problems were prepared for the 2020 ISMAA meeting which, lamentably, did not take place. We encourage **anyone or any group** of students and/or faculty to consider these problems and, if solved, write up a solution and send it to us. Think of these much like problems from the *Monthly*. In a few weeks we will publish solutions along with the names of all those who submitted correct solutions. This will be posted on the ISMAA web site. Please send solutions – with identifying names and institutions – electronically (preferably) to `pgandrews@eiu.edu` or by mail to Peter Andrews, Department of Mathematics and Computer Science, Eastern Illinois University, 600 Lincoln Ave., Charleston, IL, 61920 so that we receive then by Monday, April 11, 2022. Happy problem solving!!

1. **Splitting a Trapezoid** A trapezoid, $ABCD$, has parallel sides AB, of length b, and CD, of length a, with $a < b$. Its area is divided precisely in half by a segment of length x that is parallel to AB and CD. Find the value of x in terms of a and b.



2. **Digit Sums** Peter thinks that two or more of the six integers $2^{2020}, 2^{2021}, 2^{2022}, 2^{2023}, 2^{2024}, 2^{2025}$ have the same digit sum, while Gregory thinks all six of the digit sums are distinct. Find, with a proof, who is right, Peter or Gregory.

3. **Balancing 2020** A 2020 gram weight is placed on one pan of a simple balance. You have available an infinite set of weights, one each of weight $1, 3, 9, 27, \ldots, 3^n, \ldots$. Your assignment is to bring the scale into balance by placing some of these weights on one pan, or the other, or both. If this is possible, describe which weights go on which pan. If it is not possible, prove why.

4. **(Prime to Composite)** The integer $P = 2^{127} - 1$ is a 39-digit prime number. Find a possible digit x which, when being inserted at any place between the 17^{th} and 37^{th} digit of the decimal representation of P, converts P into a composite number. Is there more than one such a digit?

5. **April Fools** You have seven refrigerator magnets labeled $1, 2, 2, 3, 3, +, =$, respectively. You have, additionally, some more magnets that represent unknown non-zero digits $x, y, z, \ldots$. The first seven magnets are currently arranged on your fridge as $22 + 1 = 33$, which is clearly not a valid equation. You have two challenges: first, re-arrange *all seven* magnets on the fridge so that a valid equality is shown and all the seven magnets are used in that equality; second, arrange *all* the magnets you have so that a valid equality is shown. Which of the two challenges can be met and which cannot? If a particular challenge can be met, show how; if it cannot, prove why not. (You should assume there is enough room on the fridge for all the existing magnets.)

6. **A Grasshopper Hopping** A grasshopper sits at vertex A of the square $ABCD$ with side length 1. It jumps inside or on the boundary of the square, with each jump of length exactly 1. Determine and shade the set of all the points inside or on the boundary of $ABCD$ which the grasshopper can reach from the initial vertex A by making no more than 3 jumps.