Title: Invariants for Spatial Graphs  
**Authors:** Elaina Aceves, CSU Fresno; and Jennifer Elder, CSU Fresno  
**Faculty Sponsor:** Dr. Carmen Caprau, CSU Fresno  
**Abstract:** We use knot theoretic methods to construct, and study, invariants for certain type of graphs embedded in three-dimensional space.

Title: Embedding Cycles in $PG(n, q)$  
**Authors:** Elaina Aceves, CSU Fresno; David Heywood, CSU Fresno; and Ashley Klahr, University of San Diego  
**Faculty Sponsor:** Dr. Oscar Vega, CSU Fresno  
**Abstract:** Our work builds from that of Lazebnik, Mellinger, and Vega about the embedding of graphs in finite projective planes. We use their results for embedding cycles in projective planes in order to embed a cycle in planes that partition $PG(3, q)$, and then we piece those cycles together to obtain a cycle in $PG(3, q)$. We then extend that process to $PG(n, q)$.

Title: Optimal Water Use Strategies for a Growing Population  
**Author:** Rachel Bayless, Sonoma State; Micheal Cardoso, Sonoma State; and Robin Decker, Sonoma State  
**Faculty Sponsor:** Dr. Sunil Tiwari, Sonoma State  
**Abstract:** Summarizing mathematical methods and results from one of the 2013 COMAP Mathematical Contest in Modeling problems, our team presents a solution to the United States' increasing demands for freshwater. Expanding upon the traditional model of logistic population growth, we utilize a dynamic growth rate and carrying capacity to model population growth and freshwater demand. We explore several specific solutions, and examine methods to minimize economic costs and ecological impacts through linear programming, minimum spanning tree, and other methods of mathematical optimization.

Title: Investigating the Ricci flow on Riemann surfaces  
**Authors:** Derik Birdsall, Cal Poly; Michael Campbell, Cal Poly; Jason Elwood, Cal Poly; Max Garcia, Cal Poly; and Dana Hipolite, Cal Poly  
**Faculty Sponsor:** Dr. Morgan Sherman, Cal Poly  
**Abstract:** The Ricci flow, introduced by Hamilton in 1982, is a powerful tool in differential geometry used to help illustrate the structure of a manifold. Specifically the Ricci flow is a PDE defined on the metric of a manifold, and solutions — at least in principle — are expected to be metrics which “smooth out” over time. This process is often said to be analogous to how solutions to the heat equation (a classical PDE) tend to smooth out the heat distribution on an object. In our project we investigated the Ricci flow in the setting of a Riemann surface and directly compared convergence rates to that of the heat equation.
Title: Symmetry of the Numerical Range  
Author: Shelby Burnett, Cal Poly  
Faculty Sponsor: Dr. Linda J. Patton, Cal Poly  
Abstract: The numerical range of an \( n \times n \) matrix \( A \) is a convex subset of the complex plane defined by

\[
W(A) = \{(Av, v) : v \in \mathbb{C}^n, \|v\| = 1\}.
\]

The numerical range is said to have \( n \)-fold symmetry about the origin (\( n \)-SATO) if \( e^{\frac{2\pi i}{n}} z \in W(A) \) implies \( z \in W(A) \).

If \( A \) is a \( 2 \times 2 \) matrix, then \( W(A) \) is a possibly degenerate elliptic disk with foci at the eigenvalues of \( A \). While \( 3 \times 3 \) matrices have slightly more diverse numerical ranges, symmetry results for the numerical ranges of matrices of these sizes have already been largely explored and classified. Therefore, we will discuss natural generalizations of these results to \( 4 \times 4 \) matrices. We will derive a formula for a special class of \( 4 \times 4 \) matrix whose numerical range has 4-SATO and classify types of matrices whose numerical ranges produce 4-SATO.

Title: Computing Tutte Polynomials of Arithmetic Matroids  
Authors: Federico Castillo, UC Davis and Mike Henley, San Francisco State University  
Faculty Sponsor: Federico Ardila, San Francisco State University  
Abstract: Classically, a list of integer vectors gives a Matroid, but a new point of view consists of recording more information, namely the multiplicity of a subset, giving something called an Arithmetic Matroid. This is connected to hypertoric arrangements, zonotopes, and certain marked graphs. We study methods for computing very important invariants and apply them to concrete examples.

Title: An upper bound of the double lattice covering density of regular pentagons  
Authors: Peter Chang, UC Davis; and Jiahui Guan, UC Davis  
Faculty Sponsor: Dr. Greg Kuperberg, UC Davis  
Abstract: This article shows that the largest \( p \)-hexagon inscribed in a regular pentagon has area

\[
\frac{\sqrt{5} + 2\sqrt{5}}{2} \approx 1.53884
\]

where the regular pentagon is sized and has unit side length. Using this theorem, we can find the least density of a \( p \)-hexagon covering of regular pentagons, which is also an upper bound of the double-lattice covering.
Prime Analysis in Binary

Title: Prime Analysis in Binary
Author: Brandynne Cho, Saint Mary's College of California
Faculty Sponsor: Dr. Jim Sauerberg, Saint Mary's College of California
Abstract: We say that two prime numbers are "neighbors" when a one-digit change in one primes binary form produces the other. The resulting "neighborhoods" can be depicted as a graph. The graph of these primes relationships was then analyzed concerning the graphs shape, possible loops in the graph, the strength (or weakness) of the connections between primes, the size of neighborhoods, and how the number of neighbors per prime were distributed throughout the graph. Analysis of the graph prompted further questioning regarding the size of the largest group of neighbors, as well as if there are multiple large neighborhoods.

A modified Gale-Shapley Algorithm For the TA-Classes Assignment Problem

Title: A modified Gale-Shapley Algorithm For the TA-Classes Assignment Problem
Authors: Gabrielle Chwalik, UC Davis; Connor Duthie, UC Davis; Victor Fuentes, UC Davis; and Sachin Salgaonkar, UC Davis
Faculty Sponsor: Dr. Jesus De Loera, UC Davis
Abstract: We discuss an implementation of a program to solve a specific application of the Stable marriage Problem, namely assigning graduate student teaching assistants to discussion sections. Although our problem could be formulated as an integer programming problem, we instead present our matching problem using graph theory and utilizing the Gale-Shapley algorithm to generate a solution in polynomial time. Several constraints are considered, including a graduate students availability, seniority, and the need to assign a single graduate student to multiple classes.

Toeplitzness of Products of Composition Operators and their Adjoints

Title: Toeplitzness of Products of Composition Operators and their Adjoints
Authors: Matt Gagne, Cal Poly; and Chad Duna, Cal Poly
Faculty Sponsor: Dr. Jonathan Shapiro, Cal Poly
Abstract: We examine the Toeplitzness of products of composition operators and their adjoints. We show, among other things, that $C_\phi^*C_\phi$ is strongly asymptotically Toeplitz for all analytic self-maps $\phi$ of the unit disk, and that $C_\phi C_\phi^*$ is Toeplitz if and only if $\phi$ is the identity or a rotation. Also, we see that $C_\phi C_\phi^*$ can exhibit varying degrees of asymptotic Toeplitzness.

Detecting Speed Bumps in Point Cloud Data

Title: Detecting Speed Bumps in Point Cloud Data
Authors: David Goulette, San Jose State; Neeti Mittal, San Jose State; and Valerie Sui, San Jose State
Faculty Sponsor: Dr. Bern Cayco, San Jose State
Abstract: Developing fast algorithms for detecting objects in Point Cloud Data poses significant challenges due to the noise and the immense size of the data sets produced by the most powerful sensor devices. Our team developed a method for detecting parking lot speed bumps using ground plane estimations. We also developed another method which analyzes variations in local point density to detect the speed bump.
Title: Improving Particle Integration Efficiency in Mantle Convection Simulation by Combining Numerical Integration Techniques
Authors: Emily M Javan, UC Davis; and Edward H Studley, UC Davis
Faculty Sponsors: Dr. Eric M Heien, UC Davis; and Dr. Louise H Kellogg, UC Davis
Abstract: Tracer particles are often used in simulations of mantle convection and similar phenomenon to track material properties on a fine scale. Particle flow techniques generally use a single high-accuracy numerical integration method, possibly with an adaptive time step. However, convection simulations have strong heterogeneity in the types and linearity of flows. To improve efficiency we investigate exploiting this heterogeneity by using different integration methods depending on the flow pattern near each particle. The optimal integration method and time step is based on a measure of flow curvature in a given vector field, and a user specified acceptable level of error. Particles in low curvature flow tend to use Euler's method of integration, while higher curvature requires the use of second or fourth order Runge-Kutta. Our method analyzes a particle's surroundings quickly and responds with the most efficient integration method to maintain an acceptable level of error. We implement this method in the mantle convection simulation code ASPECT and find significant improvement in overall simulation performance while maintaining the same level of accuracy. We also demonstrate how this method allows a tradeoff between accuracy and speed in these simulations and how it may be applicable to other areas.

Title: The Perfect Shuffle: When shuffling can make a deck of cards less random
Author: Kevin Martin, Simpson University
Faculty Sponsor: Dr. Isaiah Lankham, Simpson University
Abstract: The perfect shuffle (aka the faro shuffle) is a version of the riffle shuffle in which a deck of cards is cut in half and the two halves are perfectly interleaved. We give an overview of the perfect shuffle and describe algebraic properties like order, meaning the smallest number of shuffles needed to return a deck of cards to its original state. We also illustrate real-world applications of perfect shuffle theory.

Title: Machine Learning of Mathematical Documents
Authors: Longphi Nguyen, UC Davis; Laila Rizvi, UC Davis; and Ying Shi, UC Davis
Faculty Sponsor: Dr. Jesus De Loera, UC Davis
Abstract: Researchers may find it difficult to identify and connect with individuals of similar interests without using extensive information. To improve connectivity, we apply machine learning methods from statistics to generate a list of terms that represents a coherent set of sentences/words that summarize the researcher's work. These methods include co-occurrence, correlation, LASSO, and L1-penalized logistic regression. By applying Sparse Principal Component Analysis, we find that we can reduce the computational time of the four methods by about 65% with little consequence to the overall results. People can use these results to network and collaborate with others of common interest.
**Title:** New Integer Sequences Arising from 3-period Folding Numbers  
**Author:** Quynh Nguyen, Santa Clara University  
**Faculty Sponsor:** Dr. Jean Pedersen, Santa Clara University  
**Abstract:** Following George Pólya's "guess and test" method, we seek to discover 3-period folding numbers analogous to the exhaustive set of 2-period folding numbers discovered by Peter Hilton and Jean Pedersen and discussed more recently in their last book, *A Mathematical Tapestry* (Cambridge University Press, 2010). In the process we discover a large set of 3-period folding numbers, which we are able to organize into a table, and show that there must be many more sets of such numbers that we have not yet found. These sets of 3-period folding numbers result in new integer sequences that are not mentioned on the Online Encyclopedia of Integer Sequences (OEIS).

**Title:** Prime Numbers Arising from Quadratic Polynomials  
**Author:** Avineet Pannu, Saint Mary's College of California  
**Faculty Sponsor:** Dr. Jim Sauerberg, Saint Mary's College of California  
**Abstract:** For \( f(1), f(2), \ldots, f(n) \), define \( S(a, b, n) \) to be the smallest possible integer \( m \), such that \( f(1), f(2), \ldots, f(n) \) are all distinct (mod \( m \)). Zhi-Wei Sun claims in his paper *A Simple Way to Generate All Primes* that he is able to generate exactly all primes using the quadratic \( f(t) = t(t - 1) \) and modulo arithmetic defined above. We use the function \( S(a, b, n) \) and define it using the coefficients of a quadratic in the form of \( f(t) = t(at + b) \) and \( n \) being the term in the sequence. Continuing Zhi-Wei Sun's research which looked at \( S(1, -1, n) \), we explore the general case where \( b \) is left as a constant, \( S(1, b, n) \). Depending on the two different cases where \( b \) is either even or odd, we see that \( S(1, b, n) \) is a combination of \( 2p, 2^h, \) and \( p \). After that, we show the results of two specific cases, \( S(2, 1, n) \) and \( S(3, 1, n) \). Interestingly \( S(2, 1, n) \) results in \( 2^h \) for \( h \) such that \( n \leq 2^h \) and \( S(3, 1, n) \) in \( 3^h \) for \( h \) such that \( n \leq 3^h \).

**Title:** Cycles and Cycle-related Graphs in \( PG(2,q) \)  
**Author:** Jamie Peabody, CSU, Fresno  
**Faculty Sponsor:** Dr. Oscar Vega, CSU, Fresno  
**Abstract:** We establish that it is possible to embed \( k \)-cycles into \( PG(2,q) \), for all \( 3 \leq k \leq q^2 + q + 1 \), as long as a hypothesis on the generators of the finite field \( GF(q) \) holds. We have verified this hypothesis to be true for finite fields of prime order \( p < 10^{10} \). Furthermore, we expand upon this result to describe the behavior of embeddings of cycle-related graphs, such as wheels and gear graphs. For these families of graphs, we have obtained sharp bounds for embeddability and ways to embed all graphs allowed by these bounds. Many of our results may be generalized to \( AG(2,q) \), and other projective planes.
Title: Pandemic at Pacific
Author: Austin Tuttle, University of the Pacific
Faculty Sponsor: Dr. John Mayberry, University of the Pacific
Abstract: The classic SIR model for disease spread utilizes a system of ODE. We compare this model with a network model for disease spread, for which we came up with four different possible degree distributions: Uniform, Power Law, Binomial, and Bimodal Binomial for. We found that the ODE model is not an accurate approximation for an outbreak when compared to any of these four distributions. We also investigated the effects of a limited number of vaccinations on each of the two of the most different networks (Binomial and Power Law). We proposed 3 possible vaccination strategies: 1. Target vaccinations to those with the highest number of unvaccinated connections. 2. Target vaccinations to those with the lowest number of unvaccinated connections. 3. Randomly vaccinate. And we found that the relative success of the first strategy compared to the other 2 was highly dependent on how the degree distribution of the population.