Mathematical Association of America
Golden Section (Northern California, Nevada and Hawaii)
Saturday, February 27, 2016
University of California, Davis
STUDENT POSTER SESSION ABSTRACTS

Title. *Dehn–Sommerville Relations and the Catalan Matroid*

Authors: Anastasia Chavez, University of California, Berkeley; Nicole Yamzon, San Francisco State University

Faculty sponsor. Dr. Federico Ardila, San Francisco State University

Abstract. The f-vector of a d-dimensional polytope P stores the number of faces of each dimension. When P is simplicial the Dehn–Sommerville relations condense the f-vector into the g-vector, which has length $\lceil \frac{d+1}{2} \rceil$. Thus, to determine the f-vector of P, we only need to know approximately half of its entries. This raises the question: Which $(\lceil \frac{d+1}{2} \rceil)$-subsets of the f-vector of a general simplicial polytope are sufficient to determine the whole f-vector? We prove that the answer is given by the bases of the Catalan matroid.

Title. *Algebraic Structures of 2-Dimensional Topological Quantum Field Theories*

Author. Joseph Dominic, Saint Mary's College of California

Faculty sponsor. Dr. Weiwei Pan, Saint Mary's College of California

Abstract. In this poster we study the mathematical aspects of 2-dimensional topological quantum field theories (2D TQFT's) culminating with a classification theorem which characterizes 2D TQFT's with a certain class of algebraic structures called Frobenius Algebras. We then attempt to construct a generalized 2-dimensional topological quantum field theory which classifies Hopf Algebras and examine the difficulties inherent to this endeavor. Finally, we study a relationship between Hopf Algebras and Frobenius Algebras.

Title. *Jacobi-Type Sums With an Explicit Evaluation Modulo Prime Powers for Small Powers of Primes*

Author. Franciska Domokos, California State University, Sacramento

Faculty sponsor. Dr. Vincent Pigno, California State University, Sacramento

Abstract. Given two mod q Dirichlet Characters χ_1 and χ_2 the classical Jacobi Sum is

$$J(\chi_1, \chi_2, q) := \sum_{x=1}^{q} \chi_1(x)\chi_2(1-x).$$

W. Zhang and W. Yao found that the classical Jacobi sum has explicit evaluations as long as q is a perfect square and Zhang & Xu obtained an evaluation of multivariate Jacobi sum for certain squareful q (if $p \mid q$, then $p^2 \mid q$). This result was extended by Alsulmi, Pigno & Pinner to the general Jacobi-Type sum

$$\sum_{\substack{x_1=1 \\ A_1 x_1^{k_1} + \cdots + A_s x_s^{k_s} \equiv B \bmod p^m}}^{p^m} \cdots \sum_{x_s=1}^{p^m} \chi_1(x_1) \cdots \chi_s(x_s), \tag{1}$$

for general squareful q using the sum reduction methods of Cochrane and Zheng as long as $m \geq 2t + n + 2$. Here B is an integer, $p \nmid A_1 A_2$ and the integers t and n are defined by

$$p^n \mid\mid B \text{ and } t = \max_i \{t_i\} \text{ where } p^{t_i} \mid\mid k_i, \ i = 1, \ldots, s.$$

In this project we evaluated the sum (1) on a smaller range $t + n + 1 < m \leq 2t + n + 2$ when $s = 2$ and p is an odd prime.

Title. *Enumerating Lattice 3-Polytopes*

Author. Mónica Gómez, University of California, Davis

Faculty sponsor. Dr. Jesús De Loera, University of California, Davis

Abstract. A lattice 3-polytope is a polytope $P \subset \mathbb{R}^3$ with integer vertices. We call size of P the number of lattice points it contains, and width of P the minimum, over all integer linear functionals f, of the length of the interval $f(P)$.

In this poster I present my results on a full enumeration of lattice 3-polytopes via their size and width: for any fixed $n \geq d+1$ there are only finitely many lattice 3-polytopes of width larger than one and size n, and this finite list can be partially obtained from the list of size $n-1$. The rest of the polytopes can be separately classified due to some special features.

Title. *A Sampling Kaczmarz-Motzkin Algorithm for Linear Feasibility*

Author. Jamie Haddock, University of California, Davis

Faculty sponsor. Dr. Jesús De Loera, University of California, Davis

Abstract. We combine two algorithmic techniques for solving systems of linear inequalities, the relaxation method of Agmon, Motzkin et al. and the randomized Kaczmarz method. In doing so, we obtain a family of algorithms that generalize and extend both techniques. While we prove similar convergence results, our computational experiments show our algorithms often vastly outperform the original methods.

Title. *Numerical Ranges of Composition Operators Whose Symbols are Disk Automorphisms*

Author. Buddy Galleti, Christopher Hurley*, and Adam Mair, Cal Poly, San Luis Obispo

Faculty sponsor. Dr. Jonathan Shapiro, Cal Poly, San Luis Obispo

Abstract. We examine properties of the numerical ranges of certain composition operators acting on the Hardy space. We pay particular attention to those composition operators whose symbols are rotations and elliptic automorphisms of the disk.

Title. *The Missing Piece: The Effect of DNA Curvature on the Topological Structure of Kinetoplast DNA Minicircle Networks*

Author. Lara Ibrahim, University of California, Davis

Faculty sponsor. Dr. Javier Arsuaga, University of California, Davis

Abstract. The kinetoplast DNA (kDNA) of Trypansoma Brucei forms a chain-mail like network of thousands of catenated minicircles. Little is known about the biophysical factors which maintain this complex network. Experimental data suggests that each minicircle is singly linked to 3 other minicircles (i.e. valence 3). However, mathematical modeling which factors in minicircle orientation, volume exclusion, and electrostatic repulsions fails to reduce the valence below 11. To account for this discrepancy between the experimental and theoretical valence number, we are analyzing the sequence dependent curvature of the minicircle sequence. We suggest that intrinsic bending or "kinking" of the DNA strand may allow for such low valence. We employ the use of the program 3DNA, which builds visualizations of the three dimensional structure of nucleic acids based on specified base pair parameters.

Title. *Algebraic Methods for Neural Codes*

Author. Nida Kazi, San Jose State University

Faculty sponsor. Dr. Elizabeth Gross, San Jose State University

Abstract. A neural code is a collection of codewords (0-1 vectors) of a given length n; it captures the co-firing patterns of a set of neurons. A neural code is convexly realizable if there exist n convex sets in some $\mathbb{R}^d$ so that each codeword in the code corresponds to a unique intersection carved out by the convex sets. There are some

methods to determine whether a neural code is convexly realizable, however, these methods do not describe how to draw a realization. In this work, we construct toric ideals from neural codes, and we show how we can use these ideals, along with the theory of inductive piercings and Euler diagrams, to draw realizations for particular classes of codes.

Title. *The Length Spectrum of the Sub-Riemannian Three-Sphere*

Author. David Klapheck, California State University, Sacramento

Faculty sponsor. Dr. Michael VanValkenburg, California State University, Sacramento

Abstract. We determine the lengths of all closed sub-Riemannian geodesics of the three-sphere S3. Our methods do not require us to directly construct solutions of the geodesic equations.

Title. *A Rainbow Ramsey Analogue of Rado's Theorem*

Author. R. N. La Haye, A. Montejano, D. Oliveros, and E. Roldn-Pensado, University of California, Davis

Faculty sponsor. Dr. Jesús De Loera, University of California, Davis

Abstract. We present a Rainbow Ramsey version of the well-known Ramsey-type theorem of Richard Rado. We use new techniques from the Geometry of Numbers. We also disprove two conjectures proposed in the literature.

Title. *Linear Systems and Chip Firing on Graphs*

Author. Bo Lin, University of California, Berkeley

Faculty sponsor. Dr. Bernd Sturmfels, University of California, Berkeley

Abstract. The linear system $|D|$ of a divisor D on a metric graph has the structure of a cell complex. We introduce the anchor divisors and anchor cells in it - they serve as the landmarks for us to compute the f-vector of the complex and find all cells in the complex. A linear system can also be identified as a tropical convex hull of rational functions. We compute the extremal generators of the tropical convex hull using the landmarks. We apply these methods to some examples - namely the canonical linear systems on K_4 and $K_{3,3}$.

Title. *The Monstrous Poster*

Author. Alexander Lowen, Saint Mary's College of California

Faculty sponsor. Dr. Kristen Beck, Saint Mary's College of California

Abstract. J. H. Conway constructs the monster group using the Leech Lattice. The purpose of my research is to exposit on the Hexacode and the extended Golay Code in order to construct the Leech Lattice. In doing so, I hope to instill an intuition for Conway's second sporadic group, Co_1, which is needed to construct the monster group.

Title. *Generalized Eulerian Numbers and the δ-polynomial for Half-Open Lattice Parallelepipeds: A Geometric Perspective*

Authors: Emily McCullough*, San Francisco State University; Dr. Katharina Jochemko, Vienna University of Technology

Faculty sponsor. Dr. Matthias Beck, San Francisco State University

Abstract. The coefficients of the δ-polynomial of a lattice polytope $\mathcal{P}$ encode information about the lattice point counts in positive integer dilates of $\mathcal{P}$. We consider the relationship between the (B, ℓ)-Eulerian numbers, a refined descent statistic on the set of signed permutations on $\{1, 2, \ldots, d\}$, and the δ-polynomial for the d-dimensional half-open ± 1-cube with ℓ non-translate facets removed. Using the interplay of geometry and combinatorics, we improve upon known inequality relations on the coefficients of the δ-polynomials for specific families of half-open parallelepipeds. Our results extend naturally to the δ-polynomials for closed lattice zonotopes.

Title. *Transportation Polytopes and their Hierarchy of Circuit Diameters*

Author. Jacob Miller, University of California, Davis

Faculty sponsor. Dr. Jesús De Loera, University of California, Davis

Abstract. The study of the diameter of the graph of polyhedra is a classical problem in the theory of linear programming. While transportation polytopes are at the core of operations research and statistics it is still open whether the Hirsch conjecture is true for general $m \times n$ transportation polytopes. In earlier work the first three authors introduced a hierarchy of variations to the notion of graph diameter in polyhedra. This hierarchy provides some interesting lower bounds for the usual graph diameter.

Here we compared the hierarchy of diameters for the $m \times n$ transportation polytopes. We show that the Hirsch conjecture bound of $m + n - 1$ is actually valid in some of these diameter notions. Second, we prove that for $3 \times n$ transportation polytopes the Hirsch conjecture holds in the classical graph diameter. Third, we show for $2 \times n$ transportation polytope that the stronger monotone Hirsch conjecture holds and improve earlier bounds on the graph diameter.

Title. *Graded Rational Fibonacci Numbers*

Author. Kirill Paramonov, University of California, Davis

Faculty sponsor. Dr. Anne Schilling, University of California, Davis

Abstract. Simultaneous (a, b)-cores (partitions with no hook lengths of a or b) are widely known in their connections with rational (a, b)-Dyck paths and rational Catalan numbers. It was discovered that the number of simultaneous $(a, a + 1)$-cores with distinct parts $\phi(a)$ is equal to the a-th Fibonacci number. We construct generalizations of Fibonacci numbers based on that fact and give a natural grading to those numbers. We then obtain recursive relations on those numbers, which turn out to be the same relations for various algebraic objects.

Title. *Studying Seifert Surfaces of Knots via Bridge Spheres*

Author. Carson Rogers, University of California, Davis

Faculty sponsor. Dr. Abigail Thompson, University of California, Davis

Abstract. Two well-known tools used to study knots in 3-dimensional space are the Seifert surfaces bounded by a knot and its bridge spheres, which decompose the knot into two untangled collections of arcs. This poster will overview a new method of using certain bridge spheres for a knot to decompose its Seifert surfaces in a special way. I will illustrate how these decompositions can be used to study the topology of Seifert surfaces under a natural equivalence relation, and how this leads to a new inequality relating the bridge number of a knot to its canonical genus.

Title. *Characterizing Singular Polynomials for Dunkl Operators*

Author. Christian Smith, University of California, Davis

Faculty sponsor. Dr. Eugene Gorsky, University of California, Davis

Abstract. We study singular polynomials for Dunkl Operators. These Dunkl Operators are significant to Calogero-Moser systems and have connections to representation theory, algebraic geometry, physics, and many other fields.

Let $f \in \mathbb{R}[x_1, x_2, \ldots, x_n]$. Define $s_{ij}(f)$ as an operator which transposes x_i and x_j in f. For instance, if $f = ax_1 + bx_2 + cx_3$, then $s_{1,3}(f) = ax_3 + bx_2 + cx_1$.

Now define the Dunkl Operator as;

$$D_i(f) = \frac{\partial f}{\partial x_i} - c \sum_{i \neq j} \frac{f - s_{ij}}{x_i - x_j}$$

where i, j span over all possible n and c is some constant.

A singular polynomial for the Dunkl Operator is a polynomial f such that $D_i(f) = 0$ for all i. Our objective is to characterize these singular polynomials and to determine their dependence on the constant c.

Thus far, we have found that for polynomials of degree 1 (i.e. $f = a_1x_1 + a_2x_2 + \cdots + a_nx_n$), f is singular if and only if $c = \frac{1}{n}$, and these degree 1 singular polynomials must obey $a_1 + a_2 + \cdots + a_n = 0$. Additionally, a polynomial in n variables of the form $W = \prod_{i<j}(x_i - x_j)$ is singular if and only if $c = \frac{1}{2}$.

Title. *A Small Grant of $100,000,000*

Author. Anthony Bardessono, Donna Martin, and Maureen Smith*, Cal Poly, San Luis Obispo

Faculty sponsor. Dr. Charles Camp, Cal Poly, San Luis Obispo

Abstract. We develop a model which rates public and private higher education institutions as potential donation recipients based on financial need of students in attendance, and the success of alumni. We select the top 10 public and private schools based on data from The Integrated Post-secondary Education Data System. Since this is a charitable distribution of funds, we use a return on investments model based on social returns for the students and institution's goals.

Title. *Some Properties of Generating Matrices for Generalized Pell Sequences*

Author. Elijah Soria, Saint Mary's College of California

Faculty sponsor. Dr. Kristen Beck, Saint Mary's College of California

Abstract. The muse for this research project are a couple of generalizations of the Pell sequence, named the k-Pell sequence and Generalized k-Pell sequence, which are defined respectively as $P_{k,n} = 2P_{k,n-1}+kP_{k,n-2},\ P_{k,0} = 0, P_{k,1} = 1$ and $G_{k,n} = 2G_{k,n-1} + kG_{k,n-2},\ G_{k,0} = G_{k,1} = a \in \mathbb{Z}^+$. The methods used to come up with new theorems for these sequences were to use $n \times n$ tridiagonal generating matrices whose determinants are equal to the n^{th} term in each respective sequence. Using these generating matrices, it was possible to make connections between certain characteristics of these matrices with the determinant, creating an accessible way to work with the sequences in question.

Title. *A Dynamic Monte Carlo Algorithm for Sampling Grid Diagrams of Knots*

Author. Shawn Witte, University of California, Davis

Faculty sponsor. Dr. Mariel Vazquez, University of California, Davis

Abstract. To sample random conformations of a specific knot is normally done through the BFACF algorithm, which samples knots in $\mathbb{Z}^3$. A similar algorithm is put forth to sample random conformations of knots in grid diagrams.

Title. *Software and Computer-based Search for Extreme Functions of the Gomory–Johnson Infinite Group Problem*

Author. Yuan Zhou, University of California, Davis

Faculty sponsor. Dr. Matthias Koeppe, University of California, Davis

Abstract. Extreme functions for the Gomory–Johnson infinite group problem serve as strong cut-generating functions for general integer linear programs. For the single-row infinite group problem, our recent work on computer based search discovers piecewise linear extreme functions with up to 28 slopes, breaking the previous record of 5 slopes due to Hildebrand (2013). Several open questions are resolved by some other newly discovered extreme functions. We discuss various search approaches, their computational performance and limits.
