Mathematical Association of America
Golden Section (Northern California, Nevada and Hawaii)
Saturday, February 23, 2019
American Institute of Mathematics
STUDENT POSTER SESSION ABSTRACTS

Title. *Parallelepiped Law of Diagonal Planes*

Author. Alana Bailey, San José State University

Faculty sponsor. Hidefumi Katsuura

Abstract. The parallelogram law of diagonals states that the sum of the squares of the lengths of the four sides of a parallelogram equals the sum of the squares of the lengths. We will translate this law into three-dimensional space by examining a parallelepiped and its different types of diagonal planes. We will define the different types of diagonal planes by the number of vertices of the parallelepiped that they contain; hence, we have 2-vertex diagonal planes, 3-vertex diagonal planes, and 4-vertex diagonal planes. We will prove that the sum of the squares of the areas of the diagonal planes is equal to the sum of the squares of the areas of the faces for the 3-vertex diagonal planes, and we will examine this relationship for the other types of diagonal planes.

Title. *Glacial Cycles and the 100 Kyr Problem*

Authors. Raymart Ballesteros and Brian Knight, Cal Poly San Luis Obispo

Faculty sponsor. Charles D. Camp

Abstract. The Earth's climate during the Late Pleistocene era (1.25 Mya to 12.5 kya) is characterized by large glacial cycles: oscillations in the size of large land-based ice sheets on a 100 kyr time scale.The underlying cause for these long cycles and the predictability of their timing remain open questions. Numerous models have been proposed to explain the observed behavior including those based on forced dynamical systems in which the observed cycles are the result of complicated interactions between the quasi-periodic astronomical forcing and internal free oscillations. The overall timing of the cycles is primarily controlled by nonlinear synchronization — a ubiquitous feature of forced dynamical systems. The strong asymmetry exhibited between the warming and cooling phases of a cycle can be associated with an asymmetry in the predictability of behavior within a glacial cycle. In the work, we explore these behaviors of forced dynamical systems using two models of the glacial cycles.

Title. *Geometric Properties of the C-Numerical Range over Diagonal Matrices*

Author. Jason Brown, Cal Poly San Luis Obispo

Faculty sponsor. Linda Patton

Abstract. Given two $n \times n$ matrices, $A, C \in M_n(C)$, we state that the C-Numerical Range of A and C is

$$W_C(A) = \{\mathrm{tr}(CUAU^*) \colon UU^* = U^*U = I\}$$

We define the subset of $W_C(A)$, were the unitary matrices are diagonal, to be $W_{\mathrm{diag}}(A, C)$.

- If A and C are 2×2 matrices, then $W_{\mathrm{diag}}(A, C)$ is an ellipse.

Title. *Steady-State Ideals*

Author. Carlos Munoz, San José State University

Faculty sponsor. Elizabeth Gross (University of Hawaii at Manoa)

Abstract. Chemical reaction network theory attempts to model the dynamical behavior of chemical reactions by studying the differential equations that arise from such reactions. Mathematically, a chemical reaction network N is a triple $(\mathcal{S}, \mathcal{C}, \mathcal{R})$ where $\mathcal{S} = \{s_1, \dots, s_n\}$ is the set of chemical species, $\mathcal{C} = \{y_1, \dots, y_m\} \in Z^{S}_{\geq 0}$ is the set of chemical complexes, and $\mathcal{R}$, the set of chemical reactions is a relation on $\mathcal{C}$ denoted by $y_i \to y_j$. According to the *law of mass action kinetics*, the change in concentration of a species is proportional to the product of concentrations of reacting species. Using this law, we can come up with a system of differential equations

$$\dot{x}_\ell = \frac{dx_\ell}{dt} = \sum_{y_i \to y_j \in R} \kappa_{ij} x^{y_i} (y_{j\ell} - y_{i\ell})$$

where κ_{ij} the proportional rate constant for $y_i \to y_j$. Given a chemical reaction netework $N = (S, C, R)$, the **steady-state ideal** $I(N) \subseteq K(\kappa)[\mathrm{x}]$ is the ideal $\langle \dot{x} \colon s_i \in S \rangle$. In our work, we are interested in when the steady-state ideal of a given reactino network is monomial and explore network operations that are ideal preserving.

Title. *Curves, Conics and Cryptography, Oh My*

Authors. Joel E. Pion and Ryan Zesch, Cal Poly San Luis Obispo

Faculty sponsor. Eric Brussel

Abstract. Elliptic curves are a deep area of mathematics with many surprising applications to real life. Pell conics are the less studied little brother of elliptic curves. Our work focused on categorizing the group structure of Pell conics over various rings. Additionally, we have taken some well known elliptic curve algorithms and converted them to work on Pell conics.

Title. *A Hopf Structure on Orbit Polytopes*

Author. Mariel Supina, University of California at Berkeley

Faculty sponsor. Federico Ardila (San Francisco State University)

Abstract. A Hopf monoid is an algebraic structure that many families of combinatorial objects share. Aguiar and Ardila (2017) described a Hopf monoid structure on generalized permutahedra. I will introduce orbit polytopes, which are generalized permutahedra that are fixed under the action of the symmetric group. I will characterize the induced Hopf structure on orbit polytopes using integer compositions, and I will present a new result relating the character group of orbit polytopes to NSym, the Hopf algebra of noncommutative symmetric functions.

Title. *Topologically Minimal Surfaces in S^3*

Author. Luis Torres, San José State University

Faculty sponsor. Marion Campisi

Abstract. In 2009, Bachman introduced the topological index of a surface S in a 3-manifold M as the minimal n such that the nth homotopy group of the so-called disk complex of S in M is nontrivial, where surfaces with well-defined index are said to be topologically minimal.

- If A is an $n \times n$ weighted shift matrix of the form $\{a_{i,j}\}$ where $a_{i,j} = 0$ if $i \neq j+1 \mod n$, then for an $n \times n$ matrix C, $W_{\text{diag}}(A, C)$ will have n-fold symmetry about the origin. In addition, the numerical radius and angles of symmetry are determined from a matrix of products from A and C.

Title. *Blue Red Hackenbush Spiders*

Author. Ravi Cho, San José State University

Faculty sponsor. Tim Hsu

Abstract. Blue-Red Hackenbush is a combinatorial game played on a graph of red and blue edges attached to the "ground". Players take turns removing a single edge of their color, also removing any other edges no longer attached to the ground; the first player unable to move loses. Understanding all possible Hackenbush problems is NP-hard, making even the study of particular classes of positions mathematically interesting.

We consider the class of Hackenbush spiders, defined as follows: A Hackenbush string is a sequence of blue and red edges connected linearly, with one end touching the ground, and a spider is a collection of strings (called, in this context, legs) with their tops meeting at a common point we call the body. We have made significant progress towards understanding spiders in special cases (e.g., all legs have length 2), and have proven some useful initial results. Based on computer experiments, we also have a conjectured best strategy (always play closest to the body) in the case where all legs have equal length, and we have found counterexamples that show that the situation is more complicated when there are legs of different lengths.

Title. *L-Functions of Graph Covers*

Author. Kyle Hammer, California State University, Chico

Faculty sponsor. Thomas Mattman

Abstract. We study the Artin-Ihara L-functions associated to Galois covers of graphs. We expand on the theory initially presented by Ihara, Bass, and Terras. We use the class number formula to give a new pure graph theoretical result about the number of spanning trees of graphs in normal covers with a certain Galois group.

Title. *Discrete Volume of Coxeter Permutahedra*

Author. Jodi McWhirter, San Francisco State University

Faculty sponsors. Federico Ardila, Matthias Beck

Abstract. The Ehrhart polynomial counts lattice points in a dilated lattice polytope. The Ehrhart polynomials of permutahedra of types A, B, C, and D have been calculated by Federico Ardila, Federico Castillo, and Michael Henley (2015). However, when a type B permutahedron is shifted so that its center is the origin, it becomes a half-integral polytope, and its Ehrhart quasipolynomial was previously unknown; the same is true of odd-dimension type A permutahedra. We use signed graphs that arise from the generating vectors of each permutahedron to determine which sets of vectors are linearly independent and thus which form parallelepipeds that are a part of a zonotopal decomposition, as well as which of these parallelepipeds stays on the lattice when the permutahedron is shifted. This yields new approaches/formulas for Ehrhart quasipolynomials for these rational permutahedra.

Topologically minimal surfaces generalize well-known classes of surfaces such as incompressible, strongly irreducible, and critical surfaces, but examples of topologically minimal surfaces with high index have only been exhibited through specific artificial constructions. In this work, we exhibit simple and natural examples of high index surfaces by showing that the disk complex of a (possibly punctured) Heegaard surface in S^3 has the homotopy type of a wedge of spheres, allowing us to precisely state its index.

Title. *Jupiter's Funky Field Lines: Spherical Harmonic Modeling of the Jovian Dynamo*

Author. Will Tran, California State University, East Bay

Faculty sponsor. Jesus Oliver

Abstract. NASA's JUNO Project launched in early 2016 and entered Jupiter's orbit on July 4, 2016. After a year of data collection, NASA's scientists presented new data and are in the process of updating Jupiter's Magnetic Field models. This poster explores the spherical harmonic model behind the original baseline JRM09 (Jovian Reference Model), as well as modeling techniques behind NASA's recent interim updates to JRM09.
