Mathematics, Music and the Arts: Making Finite Math Relevant to the Arts Major

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I have taught mathematics at Stetson University, a small comprehensive university in Florida, since 1990. Stetson is composed of three colleges; one for liberal arts and science, one for music and one for business. All students at Stetson, whatever their college, are required to demonstrate mathematics proficiency either by passing an exemption test which covers precalculus, or by taking and passing a mathematics course. Since our students come from a wide variety of backgrounds, it is often challenging to find mathematics which will be interesting and relevant.

While students in business and science have defined course sequences in mathematics, students in other disciplines do not, and so they are free to take any one of a variety of courses offered by the department to satisfy their mathematics requirement. Over the past five years, the department has developed a number of specific area courses based on the interests of faculty and students. These include cryptology, game theory, chaos and fractals, and great ideas in mathematics. Although many students opt for one of these courses to satisfy their mathematics requirement, we still offer at least two sections of the more general finite mathematics course each year (one in the fall term and one in the spring). Often we will have as many as two sections a semester, with 25 to 30 students in each. So the plurality of non-science and business students are still served by the generic finite math course for their college requirement.

Over the past two years I have taught finite mathematics three times, and I have noticed that in particular music, art, theatre and English students seem to be attracted to the course. In light of this, I have been looking for ways to add applications and mathematical topics in this course which will be relevant to them. I consider my work to be preliminary in this endeavor. Although I have discovered many new topics which I can add to the class, and developed some new examples in areas, such as probability and graph theory, which are already covered in the class, I have just started modifying the syllabus and lectures to include them. However, it is clear from my early work that there is an abundance of ways in which the finite mathematics course may be made more relevant and exciting for the arts major.

I will discuss three specific ways in which the finite mathematics course can be modified with this purpose in mind. First, one can modify the topics in the current syllabus by the inclusion of applications that are relevant to music and the arts. In particular, I have developed a graph theoretical model for analyzing plays that should be of interest to many students. Second, mathematics is used in many ways in the generation and analysis of music, and adding a section on this to the existing course may prove desirable. Third, mathematics is heavily used in art for representing and modifying figures, so discussing this in class may also prove helpful..

Revising the Current Syllabus

Several texts are being used currently at Stetson in the teaching of finite mathematics, and hence different topics may be covered in a class. However, the following shows a general outline of topics I have used in order to teach finite math over the past three years.

- Matrices and Matrix Algebra
- Graph Theory
- Systems of Linear Equations
- Linear Programming
- Probability
- Markov Chains
- Statistics

If one desires to include more applications relevant to the arts major in a finite mathematics course with the minimum amount of change, the first possibility is to look at the existing topics covered, see how they might relate to music and the arts, and to insert applications and examples which make use of situations familiar to an arts student. I have found that the students in a course are often very helpful in this themselves, and by "creating" mathematics, even in a small sense, many of them are able to start to overcome their anxiety regarding the subject.

For example, an early topic covered is the definition of a matrix with examples of its use. While a businessman might be interested in the usefulness of a matrix in recording financial transactions, such as a doctor recording patient visits and payments, a musician or artist does not care about this application. However, when this student realizes that a matrix can be used to record a schedule of lessons and practice sessions, their duration, and what was covered, it becomes much more relevant.

Another use of matrices, pointed out to me by a student, is that musical notation itself is a form of a matrix! The staff lines surely denote rows, and the notes are arranged in columns. The time signature can be used to give the "size" of each matrix form of which a piece is composed.

Probability is also a fruitful area for including more material that an arts student might find relevant. When studying combinatorics, for example, a common problem concerns the number of committees of size r persons that may be formed from a group of n. The answer is, of course, $\binom{n}{r}$. Although a student in the social sciences may find this example relevant, and many of us use it in our teaching simply because we are so familiar with it, a more artist-friendly example might be, "In how many ways can *r* extras be cast from a group of *n* people auditioning?"

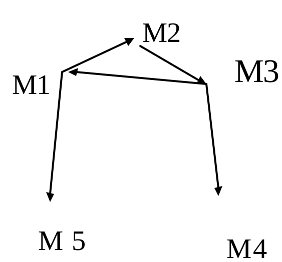
For an example with permutations, consider a group of actors auditioning for *Hamlet*. In how many ways can the parts of Laertes and Hamlet be cast? Students intuitively can see the difference between order being relevant or not in these two cases. Since this concept is often difficult even for students with stronger mathematical aptitudes, I have found that including these two examples helps to explain this idea in a more clear-cut way.

Other similar examples can be designed using a choice of recital pieces for musicians. Also, the probability of being selected as a violinist in an orchestra or as an actor in a given production provides an extension of the examples above from combinatorics to probability. Finally, how an artist arranges paintings and / or sculptures for a display may be another idea worth investigating.

The final addition to the current syllabus that I wish to address involves an application of graph theory to sociology. We can start by asking a group of people to fill out a form giving their names and the person whose opinion each values most. An example of such a survey appears below.

Name	Person whose opinion you value most
M1	M2
M2	M3
M3	M1
M4	M3
M5	M1

Now we ask, can we determine which person is dominant (exerts the most influence) in the group? Graph theory can be helpful here. We construct a digraph whose nodes are the group members and whose arcs are given by: there is an arc from *i* to *j* if M*j* values M*i*'s opinion most. So for the above survey, the digraph would be



Although it may not be possible to determine who exerts the most influence simply by looking at the digraph, a little analysis reveals the answer. We note that since M4 and

M5 have no others who most value their opinions (or whom they influence most), neither can be the most influential member in the group. We assume that a person exerts more influence the shorter the path, so that M3 exerts more influence on M1 than M2 does (since M2 influences M1 via a link to M3). Then looking at the paths shown in the digraph, we see that M3 and M1 both exert direct influence on two others: M1 on M2 and M5, and M3 on M4 and M1. So far they both seem equal. However, looking at paths of length two we note that M3 can influence both M5 and M2 via one intermediary, thereby covering all the other members of the group. M1 on the other hand can only influence M3 via a two-path, and hence has only a 3-path as a connection to M4. Thus it is obvious that M3 is the most influential group member.

This idea, while well known, can be extended to the analysis of literature. Suppose we consider a scene in a play. Let the characters in the scene be the nodes of the digraph or graph. Draw an arc between two characters if character i talks to character *j*. Then by using the above ideas, we can see who is the most influential or central character in the scene.

If all the arcs are two-way, which will likely often be the case (one character speaks to another who then replies), we can also use the ideas of shortest-chain matrices and the beta index from graph theory in the analysis.

I illustrate the ideas below using Hamlet, Act I, Scene II. The graph is shown below (Figure 1), and I note that it is a graph as opposed to a digraph since all communication is two-way. Based on an analysis using paths as above it is clear that Hamlet himself is the main character, or focus, of this scene! Since the action in the scene results in a graph, we can obtain the shortest- chain matrix. This is:

	M,B	F	Ho	Ham	K	Q	L	C,V	P
M,B	0	2	1	1	2	2	3	3	3
F	2	0	2	1	2	2	3	3	3
Ho	1	2	0	1	2	2	3	3	3
Ham	1	1	1	0	1	1	2	2	2
K	2	2	2	1	0	2	1	1	1
Q	2	2	2	1	2	0	2	2	2
L	3	3	3	2	1	2	0	2	2
C,V	3	3	3	2	1	2	2	0	2
P	3	3	3	2	1	2	2	2	0

Recalling that the row sum gives the total distance to all the other vertices, and that the smaller the sum the better connected a vertex is, we note that Hamlet (row 4, *Ham*), has a total distance of 11. The next best-connected vertex belongs to the king, at 12. So clearly, in this scene at least, Hamlet is the "best connected" or most central person, at least in this particular scene! Whether these ideas can be extended to a deeper analysis of the relation of graph theory and theatre is unclear, and research on this topic is currently in progress. However, it is hoped that this idea will at least be able to encourage English and theatre students to relate more easily to the concepts of graph theory.

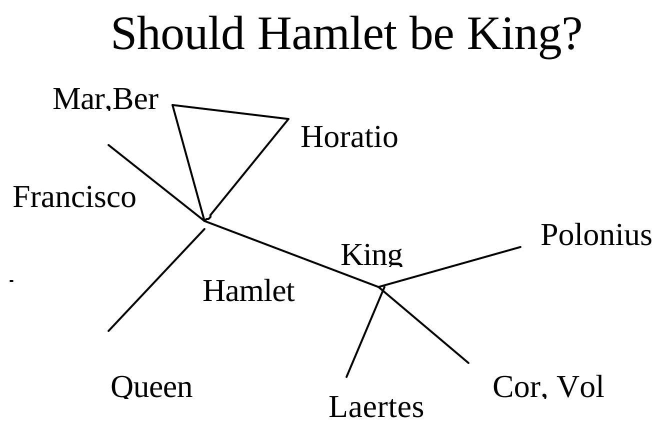


Figure 1: A graph-theoretical representation of a scene from *Hamlet*.

Mathematics and Music

Both mathematicians and musicians have long recognized the link between the two disciplines. However, it is often challenging to demonstrate this link to people who may be primarily interested in only *one* of the two subjects. Since I am concerned primarily with students of music who are taking a finite mathematics course, I will focus on ways in which music may be "mathematicized," allowing aspiring musicians to appreciate the relevance of mathematics to their major field of study.

The following represent some elements that might be added to a finite mathematics syllabus and covered in a section on mathematics and music. Below I will discuss some ideas from each major topic; exactly what topics are covered and in how much detail is, of course, up to the instructor.

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New Section: Mathematics and Music

- Sound, Waves and Pitch
- The Relationship between Pitches: Consonance, Dissonance, and Temperament
- Function and music: Exponential growth and Equal Temperament, Logarithms and Decibels

Sound, Waves and Pitch

A section on sound and waves will focus on the physics of sound. Vibrations cause sound, and in the case of music these vibrations are generally made by vibrating reeds, strings, or columns of air.

The vibrations made by a sound source produce sound waves. Important characteristics of any wave include the wavelength, frequency and amplitude. The amplitude of a sound wave will determine its loudness, certainly an important characteristic of music. *Pitch*, or how "high" or "low" in tone a sound is, comes from the frequency of the wave, measured in the number of wavelengths per unit time, or cycles per second (cps). The most common unit used in measuring frequency is the Hertz, where 1 Hertz (Hz) = 1 cps. So the pitch of a sound is determined by the measure in Hertz of the corresponding sound waves.

The higher the frequency of the sound wave, the higher its pitch, and vice versa. Humans and other animals perceive pitch differently. A human can generally hear sounds ranging in pitch from 20 to 20,000 Hz. Dogs, cats and dolphins however, can hear much higher sounds than these. In particular, it has been shown that dolphins can hear sounds up to 200,000 Hz. Elephants, on the other hand, can hear sounds in the infrasonic range below 20 Hz down to 5 Hz. The note "A" above middle "C" has a frequency of 440 Hz in most tunings, giving some measure of comparison for the facts noted above. Below we discuss the frequency of other musical sounds, which should give additional points of reference.

The first people credited with a true understanding of pitch were Galileo and Mersenne in the early 1600's. A class or individual project regarding either these two or the scientist Hertz, for whom the unit of frequency is named, might be appropriate in enhancing student appreciation of this topic. A good source for biographies of mathematicians on the web is http://www-history.mcs.st-and.ac.uk/history/.

The Relationship between Pitches: Consonance, Dissonance, and Temperament

Generally when we consider musical sounds, we think of the blending of instruments and / or voices. Usually this involves sounds made at a number of different frequencies, so how these frequencies blend is important to the making and enjoyment (or lack thereof) of music.

Humans generally perceive complex waves if two sounds with a difference in pitch greater than 7 Hz are played This is actually the result of the superposition of the sound waves, i.e. the waves “add up” or “cancel out.” In addition to blending sounds, the graphical visualization of waves via transparencies, calculators or computers can aid significantly in clarifying how this works.

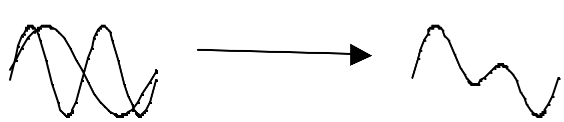

Figure 2: Waves combine to form complex sounds.

The ratios of the frequencies of sound waves involved determine whether the result is pleasant to hear or not (consonant or dissonant). The first known people to investigate what blending of frequencies would be pleasing were the Greeks. They determined that waves with a frequency ratio of 2:1 were particularly pleasing. Now we call two notes with this ratio of pitches an octave.

In modern music middle C generally has a frequency of 260 Hz. So the C an octave above would have a frequency of 520 = 2 x 260 Hz, and the C an octave lower, often called "low C" on a piano, would have frequency of 130 = ½ (260) Hz.

Another pleasing ratio of tones discovered by the Greeks was the perfect fifth. Notes a fifth apart have frequencies in a ratio of 3:2. Hence the fifth above middle C, which is the note G, should have a frequency of 3/2 x 260 Hz = 390 Hz. The fact is that as modern tunings are done, this is not quite accurate, which brings us to the notion of the 12-tone scale and temperament.

If we consider the musical scale starting at C, and going up by half –steps to the C an octave higher, there are 12 notes. These are: C, C#, D, D#, E, F, F#, G, G#, A, A#, B and C (one octave higher.)

All musicians would agree that the two C's must have a frequency ratio of 2:1. But how the notes in-between are spaced is a bigger problem. Musicians call the relationships between the tunings of the notes in a scale *temperament*. Up until relatively recently, because musicians tended to play only in certain keys and either as soloists or in small groups, the tunings in scales would vary. Now, there is a standardization of the tuning in a scale known as *equal temperament*.

The way in which an equally tempered scale is tuned is easy to see. If we consider middle C as the base note in the scale, and assign it a pitch of 260 Hz, then the note high C, the octave, has a frequency of 520 Hz. Note there are 11 equally spaced tones between these two notes, so each one must be in a ratio of $\sqrt[12]{2}$ (the twelfth root of 2) to the preceding one. This will ensure that the notes are equally spaced in pitch, and that when we get to the octave an exact 2:1 ratio of frequencies occurs, since $(\sqrt[12]{2})^{12} = 2$.

The notes of the scale from C to C discussed above will therefore have the following frequencies if tuned using equal temperament. To get each pitch, we multiply the preceding one by $\sqrt[12]{2}$.

Note Name	Pitch (in Hz)
C	260
C#	275.46
D	291.84
D#	309.19
E	327.58
F	347.06
F#	367.70
G	389.56
G#	412.72
A	437.27
A#	463.27
B	490.81
C	520

This solution and definition of the notes of a scale seems very straightforward. Why then is there a problem with equal temperament or was this way of tuning ever open to question? Although a complete answer to this question is beyond an introductory mathematics course, one reason can be easily demonstrated if we consider our earlier comments about perfect fifths.

A perfect fifth has a ratio of pitches of 3:2 with the tonic of the scale. So C and G (the fifth above C) should be in this ratio if we compare their frequencies. However, this is clearly not so: 389.56/260 = 1.49831, not 3/2. Although the actual ratio 1.49831 is very close to 3/2 or 1.5, there is a slight difference.

But is such a small discrepancy really important? Musicians look at this question via the "circle of fifths." We start with any note, say middle C, and step upward in intervals of a *perfect* fifth until we are at the same note again, but several octaves higher. It will take twelve such steps to get back to the original note, in our case a C. Note since we are using the perfect fifth, we go up by a ratio of 3/2 each time. So after 12 steps we find that the C obtained has frequency 260*(3/2)^12. For the note to be a C, it should have a pitch that is a ratio of 2 to some power with the other C. However, this is clearly not the case! Instead of a frequency of 33280, which would make the higher note a perfect octave of middle, C, the note has a frequency of 33734 Hz., off significantly. So if an instrument were tuned to insure that all fifths were "perfect," the notes in the higher octaves would sound out of tune.

Additional topics related to the ones discussed above involve scales that are not 12-tone, as is sometimes the case in modern or non-Western music, and temperaments that aren't equal. There is a great deal of information on all of these subjects available online, so investigating these questions can lead to interesting student projects.

Functions and Music

Mathematicians will of course recognize that we are already dealing with functions when we have discussed music. Sound waves can be represented by graphs of sine (and other) waves, and the pitches of the musical scale are determined by means of an exponential function. In addition, the volume of sound is measured by means of decibels, which are based on a logarithmic function.

First, let us consider the notes of a scale, as determined by the equal temperament tuning discussed above. So if we have the 12 tones of the scale equally spaced as we discussed previously, we can compute the pitches of any other notes we desire if we set the pitch of one note as the base of the scale. Using middle C, at 260 Hz, the note n tones up has a pitch of

$$Q(n) = 260 \times (\sqrt[12]{2})^{n}$$

Thus an F (n=5) has a pitch of

$$Q(5) = 260 \times (\sqrt[12]{2})^{5} = 347.06 \text{ Hz}$$

A G, the fifth, is seven half steps up. In this case n=7, and we have:

$$\mathrm{Q}(7) = 260(\sqrt[12]{2})^{7} = 260(1.49831) = 389.56 \text{ Hz}$$

As we would expect, the pitch agrees with what we computed earlier by multiplying each preceding note by $\sqrt[12]{2}$ repeatedly. Of course, if we just want the pitch of the G, the procedure that uses the function is much faster!

Another way in which music uses functions is in measuring volumes, by means of the decibel scale. Here we aren't just talking about music of course; any sound has a "loudness" or intensity which can be characterized using decibels. What is interesting from a mathematical point of view is that the decibel scale is based on logarithms. It is defined as:

$$\text{Intensity in dB (decibels)} = 10 \log_{10} (I/I_0)$$

where I_0 is an intensity of 10^{-12} watts/ square meter, the faintest sound humans can hear. As an example, a whisper has an intensity of $I = 10^{-9}$ watt/m^2. So its loudness measured in decibels is:

$$10 \log_{10} (10^{-9}/(10^{-12}) = 30 \text{ dB}$$

We can contrast this with a diesel truck at 25 feet, which has $I = 10^{-3}$ watt/m^2 Its intensity registers as:

$$10 \log_{10} (10^{-3}/(10^{-12}) = 90 \text{ dB}$$

The scale is such that successive multiples of 10 are *10 times* as intense in volume. So a truck, at 90 dB, is *1,000,000 or 10^6 times as intense* as a whisper! Another logarithmic scale commonly used, the Richter scale for earthquake intensity, works similarly (though without the factor of 10 out front.) Therefore, reminding students of their familiarity with it, or going over the math involved as an additional example, can be beneficial.

As for decibels, we note that pains starts at 120 dB, and at 150 dB ear bones will break. This is 10^{15} times the softest sound that is perceptible to humans!

Mathematics and Art

There are numerous uses of mathematics in art. Proportion, symmetry, perspective and perspective drawing are just a few applications which can be explored. It would be impossible to give a complete survey of all the connections between mathematics and the visual arts in this paper, so I will examine just two in some detail and give references in the bibliography which explore some, though hardly all, of the rest.

I will first look at the notion of proportion and the golden ratio, which has been used in the visual arts since ancient times, and then consider matrix transformations and some of their applications in the much more modern area of computer graphics.

Proportion and the Golden Ratio

One goal of art is arguably the representation of images in a pleasing or harmonious way. Thus artists are concerned with *proportion*, or the harmonious relation of the parts of some work to each other or the whole. For millennia it has been asserted that the most "pleasing" proportion, or ratio of lengths, is the *golden ratio*.

The derivation of the golden ratio is as follows. We consider a line segment divided into parts as shown in the figure below. In order for the portions of the line segment to be in the desired ratio one to another, we require the ratio of the long piece AG to the short piece GB to be equal to the ratio of the total AB to the long piece AG. So:

$$\frac{AG}{GB} = \frac{AB}{AG} \text{ or } \frac{\mathrm{x}}{1} = \frac{1+x}{x}$$

which gives

$$x^2 = x + 1$$

So

$$x = \frac{1 \pm \sqrt{5}}{2}$$

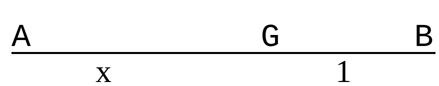

Figure 3: Illustration of computing the golden mean.

We call the larger root here, $\Phi = x = \frac{1+\sqrt{5}}{2}$ the *golden ratio* or Phi. It is approximately equal to 1/618. The other root of this equation, often called phi, is .618 approximately.

The amazing thing about this particular number is its ubiquity in applications to art throughout history. We know that its use and derivation dates back at least to the Classical Greeks. The Pythagoreans, followers of the mathematician and pre-Socratic philosopher Pythagoras, used as their symbol a five-sided star inscribed in a circle called a pentagram. The ratio of the sides of the star to the pentagon at its center was Φ. Later, the geometer Euclid had a proposition in the *Elements* showing how to draw a line with the golden ratio. The ratio of the height and the length of the Parthenon, the Greek temple to Athena, is the golden ratio. There are also many "golden rectangles"- rectangles with length to width ratios equal to an integral multiple of Φ - inlaid in the floor of the Parthenon. The Greek sculptor Phidias was involved in the building and adornment of this temple, and the first letter of his name, Phi (the Greek letter Φ), is the symbol now used for the golden ratio. Many of Phidias's other sculptures were constructed using the golden ratio as well.

The Greeks were not the only people who used the golden section in their artwork. The ancient Egyptians apparently also used the golden ratio. The Great Pyramid in Gaza may have been built by choosing its proportions using the golden ratio. Renaissance artists were fascinated with the golden ratio. Leonardo da Vinci, for example, used it in his unfinished painting *St. Jerome.* In the painting St. Jerome is inside a golden rectangle. More modern artists have made use of the golden ratio as well. For example, George Seurat, a French impressionist, used the golden ratio on every canvas he painted.

Phi is also used often in less elevated settings. Cereal boxes, posters and windows often are constructed to have the proportions of the golden rectangle. Having students measure some of these common items and compare their dimensions to the golden ratio can be a fun activity. But is Φ really the most pleasing proportion to us? George Fechner, a German psychologist, studied this question. He had subjects look at a variety of rectangles to see which ones they preferred. 75 % of participants in his study chose rectangles very close to golden rectangles (35 % exactly; another 40 % close).

John Putz in *Mathematics Magazine* makes another interesting conjecture about the golden ratio. He looked at the lengths (in measures) of the first and second parts of the movements of the 19 Mozart piano sonatas. His conjecture, which appears valid, is that the ratio of the length of the entire movement to the longer part, was Φ! This leaves open the question of whether Mozart, an undisputed genius, used the golden ratio in his compositions since it was thought to be "pleasing"?

For a more skeptical view of some assertions about the golden ratio, students could read and comment on "Misconceptions about the Golden Ratio," George Markowsky, *College Math. Journal*, vol. 23, No. 1, 1992

Matrix Transformations and Art

One final way to illustrate the connections between art and mathematics is by examining matrix transformations and graphics. Much of art, of course, involves the transformation of figures: translations, rotations, stretching or shrinking, and reflections. In mathematics, we can use matrices to perform these operations, by using matrix multiplication and / or addition.

If we let (x,y) be a point in the plane, (or (x,y,z) be a point in space), then when we multiply this point by the 2×2 matrix A, it is transformed to a new point given by A$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$.. If we consider all the points of a figure in this way, then by matrix multiplication the figure is transformed to a new one whose shape may be quite different.

To illustrate this, consider a rectangle or triangle in the plane, or any other figure that can be defined by the positions of its corners. When we multiply the points which represent the corners by the matrix A they are moved to new positions. Graphing the resulting points, and connecting them to form the transformed figure, completes the process. In order to help students visualize what is occurring, we can use Mathematica or another computer program, or even chalk and blackboard.

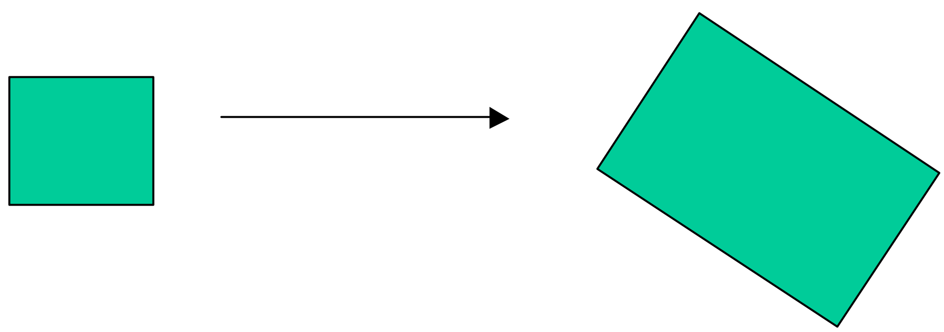

Figure 4: Matrix Transformations in Art

One important question about matrix transformations is which matrices will effect what changes on a figure? Although the complete answer to this question is most likely beyond a finite math student, some transformations are easy to learn. For example, to stretch or shrink a figure, multiply by $A = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$. If a > 1, then the figure will be stretched; if a < 1 then it will be shrunk. To translate a figure, we add the vector by which we wish to translate it to the reference points. So translating the point $\begin{pmatrix} x \\ y \end{pmatrix}$ by amount $\begin{pmatrix} a \\ b \end{pmatrix}$ is done simply by matrix (or vector) addition: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$. Finally, if we multiply by $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, a figure is rotated through an angle of θ. In general, any nonsingular matrix A (that is, a matrix which possesses an inverse) can be used to transform a figure.

The inverse problem, that of determining what matrix A produced a given transformed figure, is more difficult of course. We can assume that we know the vectors x and y for the positions of the corners of the original and transformed figures. Given them we are seeking A so that Ax=y. Although this may seem complicated, given that in the plane A is a 2×2 matrix, the resulting problem is not in fact too hard. Whether one wants to work through the mathematics with a liberal arts class probably depends on the level of the students that happen to be in it, and may differ from situation to situation. However, if we have done systems of 2 equations in 2 unknowns it is really just an extension of these ideas.

But though we can show how matrices can be used to transform figures, a curious student may ask if this is in fact ever done? Do artists employ matrices to rotate a figure through a given angle, for example, or do they just do this transformation intuitively? The answer, of course, is that with the use of computers and computer drawing programs matrices are in fact used to transform figures quite frequently, whether this is obvious to the artist or user of the program or not. The linear transformations desired must at least be specified in the source code for the drawing program to work correctly.

Conclusion

In conclusion, it is clear that mathematics and the arts have many more connections then many students, and even many mathematics instructors, may first recognize. Obviously, if one is teaching a liberal arts or finite mathematics class that has many music, art, English, theater and other similar majors in it, making the subject relevant to these students is crucial. Many students in these disciplines are anxious about mathematics to start with, and seeing mathematical applications in areas that interest them may overcome their initial reluctance in the class, and hopefully motivate them to succeed in ways that other topics wouldn't.

I have discussed applications of mathematics in music and art, and also considered ways that we might revise the current finite mathematics syllabus to include more relevant applications in topics that are already covered. Many of these ideas are preliminary, and I am in the process of developing lesson plans that incorporate them and of class testing. Preliminary responses from students have been favorable, and in fact I have been led to more applications of mathematics in the arts by suggestions from students in my finite mathematics classes. I plan on continuing to develop and write on these topics in the future, as well as incorporating any new links between mathematics and the arts I might find.

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- Viewpoints: Lessons in Mathematics and Art, http://php.indiana.edu/~mathart/viewpoints/lessons/. This site contains lessons developed by college faculty in workshops on mathematics and art, ad considers many topics not discussed in this article.
- The International Society for the Arts, Mathematics, and Architecture, http://www.isama.org/