Calculus Concepts and Technology: A Preliminary Report

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Many have been active in calculus reform for more than a decade. We began with the belief that something was wrong; the curriculum was overloaded and too many students were failing or dropping out. Related to, but not an integral part of, the reform movement has been the increased availability of computers and graphing calculators. So, for a long time, we have been experimenting with both curricular and pedagogical issues, and we have some data about our success. Student achievement and satisfaction are improved, but the data are often anecdotal and it is not clear which variables contribute to that success. Does technology alone account for it? Would the improvements come just from the curricular changes? Perhaps the pedagogy (active learning, real-world problems) should receive the credit. We must make a start on assessing all these factors.

First we make a few assumptions. Technology is here to stay, and students are coming with ever-increasing expectations and skills. With technology, students make fewer hand calculations and thus have less practice in computational skills. To allow for instruction in the use of technology, some calculus topics will have to be abridged or even eliminated. Questions arise: Is all technology good? Do students make good use of technology? Which topics can be eliminated or decreased? Do students in a technology-rich environment become less capable of algebraic manipulation?

At the beginning of the project, we asked to what extent technology helped or hindered the gaining of understanding of important concepts of the first course in calculus. But a more important and deeper question arose: What thought processes cause students to choose their problem-solving techniques? We understand that students do not present a *tabula rasa* on which we write. If we can discern what they bring to the problem we may know better how to guide the outcome.

Project Description. To determine what are considered to be the most important concepts in the first calculus course, we polled faculty in mathematics and many of our client disciplines: physics, engineering, chemistry, marine science, and economics. Most are faculty in private and state institutions in Florida. Not surprisingly, the top three concepts were fitting a curve to data, optimization, and rates of change. We then designed problem-solving scenarios for each.

Three sections of Calculus I were involved in the study in Fall 2000. Most students worked in teams of two or three, and teams were encouraged to share duties: computer input, hand calculations, calculator work, and records of results. Each pair of students had a computer for all class meetings, and each computer was equipped with *Maple* and *Mathcad*, two computer algebra systems (CAS). Graphing calculators were required. Students were given no direction about what technology to use, nor were they directed to use any. They had, however, received instructions about using a CAS, and most were at least minimally competent with their calculators.

Data Collection. The three teachers were asked to keep logs of student work, noting the technology used as well as evidence of student difficulties and frustrations. Students were asked to keep records of all attempts to solve the problem, not just the one they considered most successful. One student on each team completed a brief questionnaire about the technology used. An independent expert conducted two focus groups, and several pairs of students were videotaped.

The Puppy. The first scenario was a problem adapted from Project CALC, a calculus reform project developed by David Smith and Lang Moore from Duke University. Students were given a table of age in days and weight in pounds for every ten days from birth to day 70, and they were asked two questions: What would you estimate the puppy weighed at day 55? Can you estimate how much the puppy would weigh at six months? The following is a brief description of the work of a number of student teams.

Weight at 55 Days. Giving no reason, Steve and Al (all names are changed) decided the data were exponential and they used exponential regression on their calculator to compute a weight of 13.12 pounds. Mike and Chris verified that the data were nearly exponential and used the formula studied in the course to calculate 13.8 pounds. Dan and Stan set up a proportion to obtain a weight of 12.65 pounds. Laura and Jack looked for a pattern, then tried to compute the average between day 50 and day 60, but finally made a paper and pencil graph. They joined in the, by now, consensus value of 13.25 pounds. Three, working together, computed the slope between the points (50, 11.5) and (60, 15.0), while Becky and Francis stated that the puppy grew 2.25 pounds every ten days, and, dividing by 2, they added this number to 11.5.

Weight at Six Months. Realizing that the puppy would not continue to grow exponentially, Steve and Al switched to linear regression on their calculator to get a weight of 41.07 pounds. Mike and Chris used their formula to obtain a weight of 365.5 pounds. They made no comment about the likelihood of this. Dan and Stan stayed with their proportions, and Laura and Jack finally found a pattern. The team of three decided that the puppy would grow linearly and made up data to fit. They then graphed their made-up data to predict that the puppy weighed 41.004 pounds at six months. Becky and Francis stayed with their growth every ten days. No team commented that extrapolating far from the data would be inappropriate.

Other Data. Student self-reports and faculty logs revealed that paper and pencil remained the tool of choice. A focus group reflected basic approval of real-world problems and working in teams, but dissatisfaction with technology was evident.

Pipeline Problem. The second scenario was taken from MAA Notes Number 30. Students were given a copy of a topographical map of a wetland area with two points marked. Point A on the northwest side of the wetland was the site of a new oil well and point B on the southeast side was the site of the existing well. The students were supplied with various constraints on the cost of pipe and the cost of laying the pipe over wetland and on normal terrain. The problem was to find a route of a pipeline from B to A that would incur the least cost.

Two hints were given: (1) make an initial simplification of the problem by enclosing the wetland in a rectangle and assuming all the area within the rectangle constituted wetland, and (2) after doing some measuring to estimate costs, consider the case in which the pipeline is laid x feet north of B on normal terrain and then across the wetland to A. The activity broke down into four phases: (1) computing the cost per foot for normal terrain and for wetland, (2) getting estimates by measuring various routes on the map, (3) obtaining a cost function, and finally (4) minimizing the cost function. The students had completed the section in the calculus text devoted to optimization.

Phases 1 and 2. All eventually were able to compute the costs per foot, but many would have happily spent the entire period measuring various routes to obtain lower and lower costs. The instructor had to intervene and force attention to the hint about the cost function.

Phase 3. Lisa S., working alone, refused to abandon her measurements. Laura and Jack found a function in x and y with no indication of the meaning of y. Lisa L. and Pete found a correct function, as did Chelsea and Chuck. Becky and Ernie found a (mostly) correct cost function, but Chris and Mike, instead of adding the cost on normal terrain to the cost over wetlands, multiplied the two expressions.

Phase 4. Lisa S, convinced there was no need for a cost function, stayed with her meticulous measurements. Laura and Jack, defeated by their inability to compute the derivative of a function of two variables, fell back on one of their measurements. Lisa L. and Pete used *Maple* to compute the derivative, but they were frightened by the complexity of the result. They then tried to have *Maple* plot the function, but with no success. Chelsea and Chuck computed the derivative by hand, made an error, and obtained $x = 5.192673677$. They seemed not to notice that this result is implausible. Becky and Ernie, with the semi-accurate cost function, made a notation about computing the derivative but did not attempt to do so. Chris and Mike, having obtained a cubic cost function, did not consider a derivative. Instead, they asked *Maple* to solve their function for x, and *Maple* yielded an answer of 0. They then tried increasingly exotic and frantic calculations.

Some Observations. Little use or incorrect use was made of technology, despite the fact that all had had extensive experience with calculators and with Maple. Only one team tried plotting the cost function. Most were daunted by the need to compute a derivative of a function containing a radical, and, when the derivative turned out to be a rational function, no team thought of examining the numerator separately from the denominator. Most failed to appreciate the important problem-solving tool of considering the simpler example of a rectangular wetland area. Indeed, many refused to use this device.

Conclusions and Questions. Some dissatisfaction with technology was evident. Is this the result of too many new things at once: calculus, college, calculator, and *Maple*? Perhaps it would be wise to defer the use of a CAS until Calculus II.

Faculty expect that students will use technology to help them visualize a problem, but they appear not to do so. Most use technology for calculations only. Indeed, some express resentment when the instructor "wastes time" showing a graph. We assuredly do not plan to abandon technology, but it would appear that we must spend more class time explaining why and what we are doing.

When we think students lack understanding, it may be that they are coming from an entirely different point of view, and they are bringing to bear on the problem a very different set of assumptions. Although we think we are presenting real-world and quite relevant problems, students often see these scenarios as no more important than the made-up problems of yore (percentage of butterfat, the man rowing down stream, throwing a golf ball off the Empire State Building, etc.) Finally, we need to involve students more closely in their education. Instead of marking items wrong, we must ask them why they proceeded the way they did. Interviews and focus groups make excellent assessment tools. As Lee Shulman pointed out, successful students are given "…plenty of opportunities to talk about how they are learning, why they are learning in these ways, why they are getting things wrong when they get them wrong and right when they get them right." [1, 151-173] Perhaps our best solution would be to make fewer assignments and build in the leisure to discuss them individually or in small groups.

Endnotes

Shulman, Lee S. "Professing the Liberal Arts," *Education and Democracy: Reimagining Liberal Learning in America.* Ed. Robert Orrill. New York: College Board Publications, 1997.