MAA – EPaDel Student Competition

Kutztown University April 1, 2017

1) Find the number of integers between 1 and 50 inclusive that are not multiples of 2 or 3.

2) Find the base in which the number seven thousand, six hundred, and forty-two is represented by the symbol 1234.

3) Find all scalars a, b, and c such that

$$a(-1, 0, 2) + b(2, 2, -2) + c(1, -2, 1) = (-6, 12, 4)$$

4) Write $\tan(\operatorname{arcsec} \frac{x}{3})$ in algebraic form.

5) 55 spies are hidden throughout the world. For maximum security, any two spies send messages to each other on a frequency designated solely for that pair. What is the total number of frequencies used by the group of 55?

6) The temperature T (in degrees Fahrenheit) of food in a freezer is $T = \frac{700}{t^2 + 4t + 10}$ where t is the time in hours. How fast is the temperature dropping after 5 hours in the freezer?

7) Define
$$g(x) = \int_0^{x^2} \cos(\pi t) dt$$
, $-\infty < x < \infty$. What is the value of $g'(-1)$?

8) Consider a 6-sided fair die. If you were to roll this die 10 times, what is the probability that you roll a 3 exactly 3 times?

9) How many subsets of $\{1, 2, 3, 4, 5, 6\}$ contain 2 and 4 but not 6.

10) For $0 < a \le b$ where a, b are real numbers, find the following limit.

$$\lim_{n \to \infty} (a^n + b^n)^{1/n}$$

11) A triple of 5 year old brothers won't wear socks unless they are all wearing socks of the same color. They share one sock drawer that contains 10 pairs of each of the following colors: black, brown, white, and blue. However, none of the pairs are folded together so one of the brothers just picks out socks one by one until he gets 3 matching pairs in the same color. If all socks are identical except in color, how many should he pull out to be certain that there are at least 3 matching pairs?

12) Solve the equation $\ln(\ln(9-x)) = 2\ln 5 + 1$ for x.

13) A regular Hershey's candy bar consists of 12 even smaller pieces/units (see below). Given an $n \times m$ Hershey's bar, how many breaks would it take to break apart the big candy bar into its individual units?



14) How many subsequential limits does the sequence $a_n = \sin(\frac{n\pi}{6})$ have?

15) Consider a blank 9×9 Sudoku grid. According to the rules of Sudoku, how many ways are there to validly place the values 1, 1, 5 on the grid if each of the three values is to be placed in either the first row or the first column?

Answers:

1) 17

2) 19

- 3) a = 6, b = 2, c = -44) $\frac{\sqrt{x^2 - 9}}{3}$ 5) $\binom{55}{2} = 1485$
- 6) dropping 3.24 degrees per hour (rate of change is -3.24)

8)
$$\binom{10}{3}(1/6)^3(5/6)^7 = \frac{390625}{2519424}$$

- 10) b
- $11) \ 21$
- 12) $9 e^{25e}$
- 13) nm-1
- 14) 7
- 15) 960