# Tangles ®[*]

**Definition:** A *simple Tangle* is a Tangle that forms a single, closed, unknotted loop.

**Definition:** A *planar Tangle* is a simple Tangle that can be made to lie flat on the table.

In what follows, we explore planar Tangles in search of patterns.

1. Begin exploring simple Tangles with 10 or fewer pieces. For each simple Tangle that you create, record the number of pieces you used. Simple Tangles can be created with four or six or more pieces. No simple Tangle can be created with one, two, three, or five pieces.

2. Which numbers of pieces result in a planar Tangle? Planar Tangles can be created from four or eight pieces.

3. Once you have more than $10$ pieces, what is the smallest number of pieces required to create a planar Tangle? The smallest number of pieces greater than $10$ that can result in a planar Tangle is $12$.

4. What is the next smallest number of pieces required to create a planar Tangle? The next smallest number is $16$.

5. Do you see a pattern in your answers to Investigations 2-4? If so, describe it.

6. Does the pattern you described in Investigation 5 account for every possible planar Tangle or can you find planar Tangles whose number of pieces does not fit with the pattern? Either explain why you believe your pattern accounts for every possible planar Tangle or find at least one example of a planar Tangle whose number of pieces does not fit with your pattern.

7. Based on your answers to Investigations 2-6, can you make a conjecture about the number of pieces that can make a planar Tangle? Explain why you think your conjecture is true. The total number of pieces used to create a Planar Tangles must be a multiple of four.

The next question one might ask is, given a specific number of pieces, how many different planar Tangle shapes can be made? Before we can answer this question, we need to agree on what "different" means.

**Definition:** Call two planar Tangles *geometrically distinct* if we cannot change one planar Tangle into the other by simply rotating or flipping it.

8. For each of the first four numbers of pieces that make planar Tangles, determine the number of geometrically distinct planar Tangle shapes that can be made. Either sketch or take a picture of each of these shapes. There is one planar Tangle with $4$ pieces, one with $8$ pieces, three with $12$ pieces, and seven with $16$ pieces.

9. Do you think there is a recognizable pattern in the number of geometrically distinct planar Tangle shapes? Explain. Although one may notice a pattern among the first four planar Tangle numbers, finding a formula for the number of geometrically distinct planar Tangle with $n$ pieces is an open problem.

10. Begin exploring geometrically distinct shapes for the next smallest planar Tangle. There are $31$ distinct planar Tangles with $20$ pieces.

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[*]This activity is adapted from Discovering the Art of Mathematics, Knot Theory by Phil Hotchkiss et al.