FACULTY CONTRIBUTING PAPERS SESSIONS ABSTRACTS

Franklin & Marshall College
14 March 2015

LOCATIONS: Stager 215, 216, & 219

Stager 215 – 1:40pm **Katie Haymaker,** Villanova University

TITLE: *Graph groups that are subgroups of V*

ABSTRACT: In this talk we will discuss certain subgroups of Thompson's group V, one of the first known examples of a finitely generated infinite simple group. A graph group is an infinite group whose relations are determined by adjacencies in a finite graph. We will define a collection of graphs with a particular forbidden subgraph, and we will show that this family is precisely the graph groups that appear as subgroups of V.

Stager 215 – 2:00pm **Wing Hong Tony Wong,** Kutztown University

TITLE: *Existence of matroids with specific size, rank, and number of bases*

ABSTRACT: Let A be a matrix with r rows and n columns, where $r \leq n$. If A has full row rank, then there exists a set of r columns in A that forms an invertible submatrix. Let b be the number of sets of r columns in A that form invertible submatrices. It is obvious that $1 \leq b \leq \binom{n}{r}$. Here comes the question: Fixing a triple of integers (n, r, b), where $0 \leq r \leq n$ and $1 \leq b \leq \binom{n}{r}$, does there exist a matrix A (over any field $\mathbb{F}$) with r rows and n columns such that exactly b sets of columns in A form invertible submatrices? This question can be generalized in the scope of matroid theory, which was asked by Dominic Welsh in the first British Combinatorial Conference at Oxford in 1969: Given a triple of integers (n, r, b), $0 < r \leq n$ and $1 \leq b \leq \binom{n}{r}$, does there exists a matroid $\mathcal{M} = (X, \mathcal{I})$ with $|X| = n$, rank r, and exactly b bases? Such a matroid would be called an (n, r, b)-matroid. Mayhew and Royle suggested that an (n, r, b)-matroid always exists except when $(n, r, b) = (6, 3, 11)$. We prove that (n, r, b)-matroids always exist if the corank $n - r$ is at most 3, with the exception of $(n, r, b) = (6, 3, 11)$. We also prove that (n, r, b)-matroids exist if rank r is big relative to the corank $n - r$. This project is a collaboration with Sin Tsun Edward Fan, a graduate student at Caltech.

Stager 215 – 2:20pm **Chris Micklewright,** Eastern University

TITLE: *The Derivative Strikes Back?*

ABSTRACT: While a Calculus sequence is fundamental to every math major, it seems that many small liberal arts schools lack the time and space needed to expose students to the generalized derivative, for functions $f : \mathbb{R}^n \to \mathbb{R}^m$. However, the generalized derivative can be used effectively to tie together ideas from other standard undergraduate courses, and to revisit / emphasize fundamental concepts (such as the derivative as a linear approximation). In this talk, I will give an introduction to the generalized derivative (in a way that is friendly for students who have seen partial derivatives and matrices). I will also share how one small liberal arts college has been able to spend meaningful time on the topic, and why I think this is valuable. Finally, I will solicit feedback from the audience as to whether and how their institutions are able to cover this material. (I am curious as to the veracity of my claim that many small schools do not make the time for this topic, and whether further conversation is warranted.)

Stager 215 – 2:40pm **Brian G. Kronenthal,** Kutztown University

TITLE: *Quadratic forms: What are they and when do they factor?*

ABSTRACT: Let $\mathbb{F}$ be a field and n a positive integer. A polynomial $Q \in \mathbb{F}[X_1, \ldots, X_n]$ of the form $Q = Q(X_1, \ldots, X_n) = \sum_{1 \leq i,j \leq n} a_{ij} X_i X_j$, where $a_{ij} = a_{ji}$ for all i and j, is called a *quadratic form.* The polynomials $X_1^2 + X_1X_2$ and $2X_3^2 + 2X_1X_2 - X_1X_3 - 4X_2X_3 - 6X_3X_4 + 3X_1X_4$ are examples of quadratic forms. Consider the problem of determining, without using a computer or calculator, whether a given quadratic form factors into the product of two linear forms. It is often highly nontrivial to derive a solution by inspection. However, we can take advantage of equivalent conditions, which we will discuss in this talk. Furthermore, we will highlight vocabulary such as "reducible," "degenerate," and "singular" that is used in the literature to describe these conditions, as well as highlight the inconsistency with which this vocabulary is applied. This talk is intended for a general audience; no prior knowledge of quadratic forms will be assumed.

Stager 216 – 1:40pm **Shannon Talbott,** Moravian College

TITLE: *On Matrices for Royalty*

ABSTRACT: Given a map of the United States, what is the smallest number of distinct colors needed to color the map in such a way that no two border states have the same color? Sounds easy! Mathematically, this is the infamous graph coloring problem, which is listed as one of Karp's twenty-one NP complete problems in his 1972 paper. This talk addresses related topics from graph theory. Specifically, for $\mathbb{P}$ a finite partially ordered set (poset), we can associate with this poset a hypergraph $\mathbf{H}_{\mathbb{P}}^c$ whose chromatic number is the so-called order dimension of the poset. Furthermore, we can associate with the poset $\mathbb{P}$ a graph, $\mathbf{G}_{\mathbb{P}}^c$, whose chromatic number is bounded above by the order dimension of $\mathbb{P}$. A well known result of Trotter and Felsner proves that $\dim(\mathbb{P}) = \chi(\mathbf{H}_{\mathbb{P}}^c) \geq \chi(\mathbf{G}_{\mathbb{P}}^c)$. Here we will discuss extensions to infinite family of graphs, which arise from posets called layered generalized crowns, whose chromatic number has a known upper bound. We describe a recent characterization of this infinite family of graphs by their adjacency matrices.

Stager 216 – 2:00pm **Barry R. Smith,** Lebanon Valley College

TITLE: *Kneading Sequences*

ABSTRACT: "Kneading" is a simple operation on finite sequences of positive integers. When you see the definition, it will be clear that the sum of the entries of a sequence is invariant under kneading. I will reveal a second, less obvious invariant that allows kneading to be seen as reducing binary quadratic forms. I will also introduce some recent conjectures concerning the lengths of kneading sequences. (The content is intended for a general audience, including undergraduates of all levels. The most sophisticated definitions used will be "continued fraction" and "binary quadratic form". Both will be introduced from scratch.)

Stager 216 – 2:20pm **Rommel G. Regis,** Saint Joseph's University

TITLE: *Revisiting Positive Independence and Positive Bases*

ABSTRACT: A set of vectors S in $\mathbb{R}^n$ is said to be positively dependent if some vector in S is a nonnegative linear combination of other vectors in S; otherwise, S is said to be positively independent. Moreover, a set of vectors S is said to positively span a subspace C of $\mathbb{R}^n$ if every element of C is a nonnegative linear combination of the elements of S. A positive basis of a subspace C is a positively independent set that positively spans C. Although the concept of a positive basis has been introduced for quite some time, it has largely been ignored by most linear algebra texts. However, this topic is becoming important in the field of mathematical optimization, particularly in the design and analysis of derivative-free algorithms. This talk presents some background on positive independence and positive bases that is accessible to an undergraduate audience. In particular, it explores which properties of spanning sets, linearly independent sets and bases carry over to positive spanning sets, positively independent sets and positive bases. For example, a linearly independent set can always be extended to a basis but a positively independent set cannot always be extended to a positive basis. In addition, this talk presents procedures for constructing positive bases and for determining when a set of vectors is positively independent and when it spans a given subspace.

Stager 216 – 2:40pm **Stephen Andrilli,** La Salle University

TITLE: *Archimedes' Quadrature of the Parabola: How Was It Proven?*

ABSTRACT: The main result from Archimedes' classic treatise, Quadrature of a Parabola, is often noted in most introductory History of Mathematics courses. Archimedes proved that the area of a parabolic segment is 4/3 times the area of a triangle having the same base and equal height. (This was the first substantial discovery concerning areas of conic sections.) Archimedes' work (24 Propositions in all) actually contains two different proofs of this result. In this talk, we outline the steps involved in the second (shorter) proof (Propositions 19-24), with only one change: replacing the double ad absurdum argument at the very conclusion with a more modern approach using infinite series. An interesting side issue to be noted is that Propositions 1, 2 and 3 are left unproven in Archimedes' work, even though Propositions 1 and 3 are needed for both of Archimedes' proofs.

Stager 219 – 1:40pm **Agashi Nwogbaga,** Wesley College

TITLE: *Challenges and Tips for Project-Based Teaching of Non-Math Majors*

ABSTRACT: Recently, a student suddenly said, "*That is exactly how I feel!*". She was pointing to a joke that someone pasted on the wall in a secretary's office. It said "*I hate math tests because all through the chapter, it's really easy and you think you got it. Then the test is like: If I throw a triangle out of the car while it is traveling 20 km/hr and wind resistance is a thing that exists, how many cupcakes can Pedro buy with one human soul?*". As teachers, we enjoy teaching because of the positive differences we make in the lives of our students. Many students work very hard even under excruciating personal circumstances. But sometimes, we encounter challenges, the chief of which may be teaching students who could succeed but do not try hard enough. In this talk, we will discuss the challenges and tips for teaching general education math courses to academically challenged students who are both under-prepared and ill-prepared students. Furthermore, we will provide practical tips on how to motivate students with project-based experiential learning and cross-disciplinary approaches as well as the challenges associated with this pedagogy. We will showcase evidence-based practices and simple strategies that can promote interdisciplinary transfer of learning and can help a wide variety of students including those who profess a strong hatred for math and are so scared of math that they struggle not to develop anxiety attacks at the mention of the word math.

Stager 219 – 2:00pm **Daniel P. Wisniewski & John T. Garey,** DeSales University

TITLE: *Pre-Service Teacher Preparation in the Student Learning Objective (SLO) Process*

ABSTRACT: The Pennsylvania Department of Education (PDE) has begun implementing a process for measuring educator effectiveness based on student achievement of content standards; in particular, PDE has created the Student Learning Objective (SLO) Process as part of a multiple-measure, comprehensive system of Educator Effectiveness (authorized by Act 82) throughout the Commonwealth of Pennsylvania. During the spring 2014 semester, in anticipation of this new procedure on the horizon, pre-service teachers (undergraduate and graduate) at DeSales University (DSU) were required to conduct an SLO Project which follows the template of this PDE assessment process. The focus of this presentation will be a brief outline of PDE's newly created form of measurement, but more importantly, concentrated attention will be given to the personal experience of completing the SLO Project by the student-teacher in the mathematics secondary classroom. Perspectives on the procedure, highlighting the mutual learning of effective pedagogy and the challenges of proper assessment of remedial, at-risk high school students, will be shared. Some reflections will be offered about including the SLO as part of the preparation of pre-service teachers at DSU, including the adaptation of instructional planning based upon data-driven assessment.

Stager 219 – 2:20pm **Eric Landquist,** Kutztown University

TITLE: *Logarithms are Hot Stuff and a New Rating Scale for Chili Peppers*

ABSTRACT: Have you ever wondered what makes a hot pepper hot and why some peppers are hotter than others? The piquancy of a pepper is caused by the chemical *capsaicin*, among other so-called *capsaicinoids*. Currently, the hotness of a pepper is measured using the Scoville scale, which is proportional to the concentration of capsaicinoids, and ranges from 0 to 16,000,000. The hottest pepper in the world, the Carolina Reaper, measures up to 2,200,000 on this scale. Such large numbers can make the Scoville scale a little cumbersome for a label on a bottle of hot sauce or for the average chili pepper enthusiast. Trigonometry and Calculus students were challenged to create a more palatable rating system for chili peppers using oft-dreaded logarithms, using the familiar Richter Scale as a motivational example. Calculus students had the added challenge of making their scale a smooth piece-wise function of the Scoville rating. We present the results from their endeavors in this talk, both with the proposal of a new and simple chili pepper rating scale and with anecdotes of how this activity has helped students better understand logarithms. As such, this talk serves as both a teaching and research talk, from helping students see that log(0) doesn't exist to why we might rate that Carolina Reaper an 8.56.