MAA - EPaDel
Student Math Competition

April 6, 2013

Dickinson College

1) Find the coefficient of x^4y^5 in the expansion of $(2x+y)^9$.

2) If two positive real numbers a and b satisfy the equation

$$\frac{a+b}{a} = \frac{a}{b}\ ,$$

find the value of the ratio $\dfrac{a}{b}$.

3) Find the value of $\tan(\sin^{-1}(\frac{1}{5}))$.

4) Find the angle θ in the interval $[\frac{\pi}{2}, \frac{3\pi}{2}]$ satisfying $(\sin\theta + \cos\theta)^2 = 2$.

5) Suppose the terms a_n of a convergent sequence $\{a_n\}$ satisfy the following recursive formula:

$$a_1 = \frac{1}{2}, \quad a_n = \frac{\frac{1}{8}}{\frac{1}{4} + a_{n-1}} \quad \text{for} \quad n > 1.$$

Find the limit of the sequence.

6) Evaluate $\displaystyle\lim_{x\to 0^+} x^{\sqrt{-\frac{1}{ln\ x}}}$.

7) Name all prime numbers between 80 and 100.

8) Find the area of the largest possible rectangle that has two of its vertices lying on the curve $y = 12 - x^2$ above the x-axis and its opposite pair of vertices lying on the x-axis.

9) Find all points (x, y) lying on the ellipse $xy + x^2 + y^2 = 1$ where the tangent line to the ellipse has slope -1.

10) If $s_n = \dfrac{n-1}{n+1}$ where s_n denotes the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$, find the general term a_n of the series.

11) For what real numbers a is the matrix invertible

$$\begin{bmatrix} 0 & 1 & a \\ a & -2 & -1 \\ -1 & a & 0 \end{bmatrix}$$

12) A bag consists of 6 red marbles and 3 blue marbles. If two marbles are randomly selected from the bag without replacement, what is the probability of picking one red marble and one blue marble?

Answers:

1) 2016

2) $\frac{1+\sqrt{5}}{2}$

3) $\frac{1}{\sqrt{24}}$

4) $\frac{5\pi}{4}$

5) $\frac{1}{4}$

6) 0

7) 83, 89, 97

8) 32

9) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), (-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$

10) $a_1 = 0$, $a_n = \frac{2}{n(n+1)}$ for $n > 1$

11) all reals except 1, $\frac{-1+\sqrt{5}}{2}$, $\frac{-1-\sqrt{5}}{2}$

12) $\frac{1}{2}$