

Abstracts of Student Talks
Mathematical Association of America
Allegheny Mountain Section Meeting
Gannon University
Friday, April 1, 2016

Domenico Andreoli, Edinboro University of Pennsylvania,
Planar orbits in Schwarzschild and Kerr-Newmann Metrics

Zurn 129, 7:35 – 7:50

The project determined a planar orbit of a particle with mass m for three different cases. The first is the Newtonian case, an orbit around a star, second is an orbit in the Schwarzschild metric, a static non-rotating black hole, and finally, the Kerr-Newman metric, a rotating, charged black hole. Each case was examined with the particle at fixed angular momentum and energy, they are multiples of the radius of the Schwarzschild radius ($2GM/c^2$). MATLAB was utilized to make plots of the radial coordinate $1/r$ versus the angular coordinate ϕ for each case. The r coordinate was scaled by dividing r by the Schwarzschild radius to interpret when the particle collapsed into the star or black hole and the coordinate ϕ was scaled by dividing by 2π to show full orbits, between 0 and 2π .

Kristina Bell, Kallie Simpson, and Kalene Ireland, Slippery Rock University
Statistical Analysis: A Pedagogical Investigation

Zurn 130, 7:55 – 8:10

In recent years, there has been a debate in the education field regarding the success of using “flipped” classrooms as a new teaching pedagogy. Flipped learning is defined by flippedlearning.org to be, “a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment...” The lack of published literature regarding this topic is hindering the improvement of the learning process. The primary purpose of our study is to perform cluster analysis on the final course grades of students from Elementary Statistics 1 taught in regular and flipped classroom settings by a single professor at Slippery Rock University to identify clusters that might give insight into the two different teaching pedagogies. The data collection took place from Fall of 2013 to Spring of 2015, where each class spanned a single semester. In addition to this investigation, we will be performing a Principle Component Analysis to examine the underlying structure of the data by reducing the number of variables and eliminating any potential multicollinearity between the variables. These classes include a variety of majors, class status, genders, and evaluation methods. We are concentrating on Hierarchical Clustering, K-Means Clustering, and Principle Component Analysis through the use of R software.

Rebekah Bright, Slippery Rock University
The Solution to Math Horizon's "Golden Root" Problem

Zurn 130, 8:15 – 8:30

Math Horizons posed a problem entitled "Golden Root". We are asked to show that the golden ratio raised to the negative n power, is one of the roots of the polynomial $x^2 + ((-1)^{(n+1)}L_n)x + (-1)^n$, where L_n denotes the well-known Lucas numbers. The Lucas numbers satisfy the same relationship as the Fibonacci numbers, except that $L_0=2$ and $L_1=1$. we use factoring and relationships between the Lucas numbers, Fibonacci numbers, and the golden ratio to show that the golden root to the negative n is a root, and to find the second root of the polynomial.

Sami Condie, Grove City College
Showing Fraud with Benford

Zurn 235, 8:35 – 8:50

Benford's Law demonstrates the fact that the frequency of the integers 1 through 10 does not follow a uniform distribution, contrary to popular belief, but instead follows the distribution of $\log(n+1)/n$. This presentation briefly illustrates how Benford's Law can be proved using the Stripey-Hat Method and the restrictions needed on a sample in order for Benford's Law to apply. Under these restrictions, we will apply real voter and accounting examples to demonstrate how Benford's Law can be used to prove fraud.

Michael Curtis, Penn State Behrend
A combinatorial result and its possible extensions

Zurn 439, 7:35 – 7:50

Factorials are a unique sequence of non-negative integers denoted by $n!$. It is the product of all positive integers less than or equal to n . The factorials have very interesting properties, they grow faster than all polynomials and they are always even at $n \geq 2$. For $n \geq 5$ they are always divisible by 10. Factorials have many applications in combinatorics, computer algorithms, calculus, and many more. One of our results have yielded an equation (1), proved inductively and directly.

$$(2n)!/n!n! = (1+n/n)(1+n/(n-1))(1+n/(n-2))(1+n/(n-3))\dots (1+n/2)(1+n/1), (1)$$

Our next steps are to delve into general formulas for

$$((2n)!)/(n!(n-k)!), 0 \leq k \leq n$$

Then we will observe how the equations react to the denominator being greater than n . Hopefully our final steps in this project will be to combine all of the formulas together into a single general formula that describes the behavior of all factorial problems of the form, $A!/B!C!$.

Brandon Eschborn, **Edinboro University of Pennsylvania**
Babylonians Tracked Jupiter

Zurn 129, **7:55 – 8:10**

Displacement using velocity versus time was previously believed to originate in 14th-century Europe. However, recently deciphered tablets from ancient Babylon now point to a similar method some 1500 years earlier. In this talk, the idea of how this was accomplished will be discussed as well as other related geometric figures.

Thomas Fisher, **Penn State Behrend**
An Analysis of the Development of Slope Across a Textbook Series

Zurn 439, **7:55 – 8:10**

Slope is one of the most important topics in the math curriculum. This presentation analyzes how slope was developed across a reform-based textbook series. The results are then considered in the context of previous research on this topic.

Andrea Flores, **Mercyhurst University**
Analyzing the Dynamics of World Income Distribution Using a Markov Transition Method

Zurn 445, **7:35 – 7:50**

A crucial question in development economics is whether poor countries are catching up to their rich counterparts in terms of income and productivity. This idea, known as economic convergence, has been the subject of a considerable amount of empirical studies and theoretical work incorporated into modern economic literature over the past decades. Economists have used methods based on transition matrices and Markov chains to address the fundamental question of convergence. While based on simplistic assumptions, these methods prove helpful in the study of income distribution dynamics across countries. This talk will introduce and apply such techniques to analyze how the distribution of income across 167 countries changed throughout the period 1990-2010, and to predict the steady state of world income distribution based on our results.

Justin Goodwill, **Duquesne University**
Image Fusion Using SURE-Guided Piecewise Linear Estimation

Zurn 235, **8:55 – 9:10**

In recent years, a number of image processing algorithms have employed the Gaussian mixture model (GMM) as a probabilistic patch-based paradigm for data classification and signal estimation, achieving near state-of-the-art results. Yu, Sapiro, and Mallat developed a general framework for solving inverse problems through the connection that the Wiener filter estimation of an image patch from a GMM is precisely equivalent to sparsely representing an image patch using an structured over-complete PCA dictionary. Wang and Morel expanded upon this work by developing a piecewise linear estimation (S-PLS) using a flexible Bayesian Gaussian factor model and a SURE (Stein's unbiased

risk estimator) guided statistical filter selection. In light of Wang and Morel's results for single image denoising, we show how the S-PLS formulation can be adapted for fusing multiple images that have been corrupted by additive Gaussian noise.

Keilah Ireland, **Westminster College**
Multiple Linear Regression vs. Partial Least Squares

Zurn 447, **8:35 – 8:50**

One of the biggest questions when creating a predictive model, is what method to use. In this presentation we will be comparing a Multiple Linear Regression (MLR) model with a Partial Least Squares (PLS) Model. Both of these models are a specific kind of Inverse Least Square (ILS) model. Based on the data set and what the desired result is, one model is generally better than the other. By looking at the process, relevant output, and the amount of variance explained by each model, one can decide on the best method for creating a model.

Kinardi Isnata, **Duquesne University**
Mathematical Techniques of Image Information Fusion

Zurn 235, **7:55 – 8:10**

When an image is acquired using a given camera setting, it can typically only capture limited information of the real-world scene due to technological limitations. One way to acquire all desired information is using image information fusion techniques. Image fusion is one of the emerging topics in the digital image processing field that enables us to extract information across different images and combine them into one image having all desired features. In this talk, we will discuss several examples in image fusion along with the mathematical algorithms used to solve them, including variational methods, wavelets, and basic tools from linear algebra.

Nancy Isner, **West Liberty University**
Generating (beautiful) curves of prescribed curvature with Excel

Zurn 445, **8:15 – 8:30**

We recall the definition of curvature of curves in the plane and then present a method of approximating curves whose curvature is prescribed as a function of arc length. The result is an iterative formula derived from taking a "differentials view" of the equation $\kappa = \left| \frac{d\phi}{ds} \right|$, where ϕ is the angle the tangent vector to the curve makes with a fixed direction. A wide variety of very attractive curves can be generated using simple curvature functions.

Amanda Kowalczyk, Westminster College

A model of network formation and foraging strategies exhibited by Physarum polycephalum

Zurn 447, 7:35 – 7:50

Slime molds are a group of unrelated organisms that all have a “slimy” stage in which many single-celled organisms cling together and form a single, gelatinous organism. One slime mold, *Physarum polycephalum*, exhibits intelligent behavior such as solving a maze; the organism grows into long tubes that form a network between food sources at the beginning and end of the maze. Based on its network growth, *Physarum* has been shown to solve shortest path problems and was used to create a new algorithm to solve such problems. Our objective was to examine both *Physarum*’s initial network formation as well as its foraging strategy after network formation. We investigated *Physarum*’s growth strategies by exposing the organism to two food sources that were of different qualities. We then quantified the organism’s network by measuring the length and width of network tubes and the network’s complexity. Complexity was determined by measuring fractal dimension, a measure of the area covered by the organism; the fractal dimension encompasses all vertices and network tubes in the organism. By measuring fractal dimension, we were able to observe *Physarum*’s overall growth and retraction as it interacted with food sources. We used our data to formulate a model to represent *Physarum*’s growth and network formation. We then compared experimental measurements and observations to our model’s predictions to evaluate the accuracy of our model. Models such as this one are the first step toward formulating bio-inspired algorithms, which can often be used to solve problems more efficiently than traditional methods.

Xueyi Lei, and Blynn L. Shideler, Washington & Jefferson College

Differential Growth models

Zurn 329, 8:55 – 9:10

We studied two first-order ordinary differential equations that can be used to model growth functions, the Logistic Model and the Gompertz Model. We investigated the history of each model and derived the growth function that satisfies each respective ordinary differential equation. In analyzing the two differential equations, we compared growth behavior properties of each model function including carrying capacity, points of inflection, and model applications.

Kyle Logan, Governor's School for Science and Technology

TSER Value Determination of NFL Quarterbacks

Zurn 447, 7:55 – 8:10

In American football, an efficient quarterback is key in scoring and winning a game. This study sought to test whether a higher quarterback TSER value could possibly determine the chance of a possible team winning record during the regular season based on the TSER Scale. This study was conducted based on ten African American Quarterbacks in the NFL based on a touchdown-to-sack efficiency equation. The ten quarterbacks selected for research were from a variety of decades from the late 1970s to the early

2010s and a sixteen game season was used as well. For each player, their efficiency mean values were calculated to determine a final efficiency value for their careers. The TSER equation was used to display the trends in how many total touchdowns were scored by the quarterbacks compared to how many times they were sacked. The higher values based on the data, were approximately around 8 to 16.2, Daunte Culpepper having the highest value. The overall data in this research was varied since the quarterback played for different teams and played with different styles of offense. This study could also be used for future determination in deciding a fixed salary for the players based on performance.

Kate Lorenzen, Juniata College
Tri-Fecta of Coloring Knots

Zurn 445, 7:55 – 8:10

A knot is a circle smoothly embedded in three-dimensional Euclidean Space. A diagram is a projection of a knot onto two-dimensional space. Certain properties of a knot diagram allow us to tell if two knot diagrams represent two distinct knots. These properties are called invariants. A Fox n -coloring is a coloring of the arcs of a knot. A Dehn n -coloring is a coloring of the regions of a knot. An Alexander-Briggs n -coloring is a coloring of the crossings of a knot. The number of non-trivial Fox, Dehn, or Alexander-Briggs n -colorings is an invariant of a knot. Carter, Silver, and Williams were able to show that there is a one to one correspondence between these three invariants and explicitly found three of the mappings. This means that these three different invariants are merely different flavors of the same invariant. We extend their work by explicitly finding the remaining mappings and show that these new maps are inverses of each other. This completes the triangle of mapping from one of these colorings to another.

James Matuk, Duquesne University
Enhancing Image Denoising with Level Line Information

Zurn 235, 8:15 – 8:30

Image denoising via patched based methods have produced state of the art results in terms of image restoration quality. Although successful, these methods lack a way to incorporate geometry into the reconstructed image and visually the results are not always ideal. Recent work by Bertalmio and Levine has given a way to incorporate geometry using the curvature of the level lines of the image function. The idea is to take an image denoised with a relatively accurate patch based algorithm and perform a post-processing step which enhances the accuracy of the curvature of the level lines. This work requires removing noise from curvature data, which we have recently found a more efficient and effective mechanism for doing so based on the curvature noise distribution. In this talk, we will first present a brief explanation of image noise and how patch based methods estimate a clean image. Then, a way for incorporating level line information into these methods will be discussed along with our recent contributions to this framework based on the statistical properties of the curvature noise. We will conclude with numerical and visual results which show consistent improvement for both grayscale and color images with respect to a state of the art patch based method.

Tyler Miller and Ciel Wu, **Washington and Jefferson College**
Remote Coin Flipping

Zurn 329, **7:55 – 8:10**

In this presentation we will introduce remote coin flipping games using mathematics. Coin flipping is a popular way to settle disputes because it offers a 50% chance for either person to win. But there are disadvantages to the simple game of just flipping a coin; if the two people are not in direct contact with each other, one of them might cheat. So is there a variation to the original coin flipping problem that can eliminate cheating? This talk presents a way to flip a coin remotely using number theory which preserves the original fairness of the game and eliminates the ability to cheat.

Lauren Minner, **Penn State Behrend**
Mathematical Arachnology: A Preliminary Study

Zurn 439, **8:15 – 8:30**

In this presentation, we discuss the preliminary results obtained from our semester-long study of "Mathematical Arachnology". That is, we discuss each of the phenomena of tarantulas (Family: Theraphosidae) for which we have applied, in some form, a mathematical model. As a byproduct, our research is intended to emphasize the fact that tarantulas are significantly understudied.

David Olszewski, **Edinboro University**
Odd Perfect Numbers

Zurn 129, **8:15 – 8:30**

I plan on presenting my work on proving that there are not any odd perfect numbers. I will be using induction as well as geometric finite series to show that given any two distinct primes greater than two, raised to any powers, will never be perfect.

Brady Sheehan, **Duquesne University**
Optimality Bounds for Recovering Geometric Information in Images

Zurn 235, **7:35 – 7:50**

Techniques for denoising natural images have seen incredible advances over the past several decades. Recent statistical analyses have shown that these algorithms are approaching optimality with respect to minimum mean squared error when denoising natural images directly. However, it is still not possible to perfectly remove noise from an image and even state of the art algorithms leave visible artifacts. This allows one to question whether mean squared error is the best 'measure of goodness'. Additionally, some have questioned whether denoising an image directly is the best approach. Rather than denoise an image directly, an increasing number of algorithms have been proposed for denoising geometric features of an image and using this new information to

reconstruct a clean image. These approaches open the door to improving other measures of goodness when removing noise from an image. In this work we are studying optimality bounds for denoising geometric features of an image with the hope of discovering if there is more potential for improvement when working within this framework. Finding optimal denoising bounds for this new framework could be a breakthrough that would allow for the development of algorithms that far surpass the state of the art when denoising natural images directly.

Kevin Shuman, **Edinboro University of Pennsylvania**
The Morley Trisector Theorem for an Isosceles Triangle

Zurn 130, 8:35 – 8:50

The Morley Trisector Theorem states that the associated Morley triangle of any triangle is equilateral. The Morley triangle is a triangle formed at the intersections of the angle trisectors of the given triangle. This proof will look at a special case when the given triangle is an isosceles triangle.

Annie Small, **Juniata College**
The Mathematics of Horseshoe Shuffling

Zurn 445, 8:35 – 8:50

The horseshoe shuffle is a newly explored way of shuffling cards. The shuffle is performed by splitting a deck of cards into two piles. The bottom pile is flipped over and the cards are interlaced in an alternating pattern. We explore the following questions: How many horseshoe shuffles does it take to recycle a deck of cards to their initial positions? What is the minimum number of shuffles to get a card to the top of the deck? What properties are unique to decks of size 2^k ? What properties do cards that are equidistant from the center of the deck share? How are cards partitioned into equivalence classes?

Denver Stahl, Ravneet Singh, and Anna Cook, **Washington & Jefferson College**
Multiple Cascades

Zurn 329, 8:35 – 8:50

A very common problem in differential equations is one involving the dissipation of a mixture throughout a cascade of two or three tanks. In an attempt to extend these types of problems into a slightly abstract realm of thinking, we considered a cascade of tanks much larger than that. Through simple induction and common techniques used to solve differential equations, a pattern will be shown that can be used for any tank in a cascade of nearly infinite tanks.

Kristine Thomas, University of Pittsburgh at Johnstown
Models of Metal Production

Zurn 447, 8:15 – 8:30

Several mathematical models are presented to determine the optimal methods of producing both metal ingots and metal alloys while meeting various predetermined specifications.

Angie Toth, Edinboro University
A Generalization of Convexity

Zurn 129, 8:35 – 8:50

We present a simple generalization of convex sets and give some results related to this new concept, including a formula that describes this generalized convexity for specific sets. We also suggest some future related undergraduate research projects.

Tyler Tracy and Dan Scutella, Penn State Behrend
Decoding Calculus Students' Conceptions of Limit

Zurn 439, 8:55 – 9:10

This presentation will detail the results from a study on college calculus students' conceptions of limit before and after instruction. Implications from this research will be addressed. We will briefly discuss pedagogical recommendations based on prominent and persistent misconceptions.

Rachael Troutman, Edinboro University of Pennsylvania
Median Concurrence Theorem: An Euclidean Exploration

Zurn 130, 7:35 – 7:50

Using Euclidean Geometry, this project demonstrates one of the various proofs of the Median Concurrence Theorem. The Median Concurrence Theorem articulates that the three medians of any given triangle have a point of concurrency. With the implementation of the Parallel Projection Theorem, three cases are considered to verify that any two medians of the triangle will intersect at a point. While compiling these three cases, the Point Construction Postulate leads to the revelation that the three medians will intersect at the same point of concurrency.

Tom Tuberson , Penn State Erie, The Behrend College
Uniting Algebra and Analysis via Lattices

Zurn 439, 8:35 – 8:50

In mathematics there are two distinct areas of study, algebra and analysis. One deals with the finite and the other deals with the infinite. A notable connection exists between the two areas through the structure of lattices. Mathematicians have discovered lattice structures in topology, commutative rings, groups, etc. In this talk, we will give an introduction to lattices and then show how this structure is present in both areas, algebra and analysis.

Leah Vaughan , Washington & Jefferson College
Palindromials

Zurn 329, 8:15 – 8:30

Palindromials, also known as self-reciprocal polynomials, will be introduced in this talk. The palindromic nature of the coefficients presents interesting properties, many of which are useful tools when working with other polynomials. In particular, certain substitutions and degree reducing algorithms with palindromials can ease factoring of higher degree polynomials. Applications of palindromials extend to mechanical vibrational analysis, proof of the Fundamental Theorem of Algebra, and coding theory. Also, unexpected trigonometric identities can be found using palindromials. Here, palindromials deserve the spotlight.

Josh Whitlinger, University of Pittsburgh at Greensburg
5-Arc Connectedness

Zurn 329, 7:35 – 7:50

A space X is n -arc connected if given n different points, there is an arc in X passing through the points. We will prove that every 3-regular graph with no cutpoints is 5-Arc Connected.

Bradley Wolfe , Edinboro University of Pennsylvania
Pascal's Theorem: A Euclidean Approach

Zurn 129, 8:55 – 9:10

We shall give a direct proof of Pascal's theorem, which states that if six arbitrary points are chosen on a circle in order to form a simple hexagon, then the three pairs of extended non-parallel opposite sides of the hexagon intersect at three points that are collinear.