Abstracts of Student Talks

Mathematical Association of America
Allegheny Mountain Section Meeting
West Virginia University, Friday April 13th, 2012

7:35-7:50 PM

Matthew Stoffregen, University of Pittsburgh, Armstrong 119
Generalizations of Covering Systems

A covering system is a set of arithmetic progressions with distinct differences for which every integer falls into at least one of the arithmetic progressions. We will discuss some classical well-known results about Covering Systems. Secondly, we will discuss Covering Systems for rings other than Z, extending conjectures about the Z case to this context. We will also give a geometric interpretation for covering systems of number fields in terms of lattices. In order to treat generalized covering systems we introduce a combinatorial object, the "tree" of a set of ideals of a ring. We prove facts for the simplest trees which hold across all rings sufficiently similar to number fields. These facts show that many of the rings behave in the same way. We conjecture that there is a certain large class of rings for which many of the covering properties of Z must hold.

Carl Parkin and Hoyt Mihalak, Edinboro University, Armstrong 206
Expanding Yahtzee: Applicable Changes With Abnormal Die

Looking into the alteration of the game Yahtzee when using die other than the standard 6-sided, and what changes to scoring, strategy, and choices to make when you play using a different die set. Used probabilities and Markov chains (transition matrices) from 4-sided die to look at changes that can be made to keep the outcome of the game relatively the same.

Karissa Vasconi, Gannon University, Armstrong 303
New Results on an Anti-Waring Problem

In a previous paper by Johnson and Laughlin, then number $N(k, r)$ was defined to be the first integer such that it and every subsequent integer can be written as the sum of r or more distinct powers of k. The main result of that paper is that $N(2, 1) = N(2, 2) = N(2, 3) = 129$. In other words, the last number that cannot be written as the sum of one or more distinct squares is 128. This work is an extension of their results. It has been found that $N(2, 4) = 129$, $N(2, 5) = 198$, $N(2, 6) = 238$, $N(2, 7) = 331$, and $N(2, 8) = 383$. Additional results have been conjectured, and research is ongoing.

Elliot Blackstone, Penn State University - Erie, Armstrong 306
Generating Functions for the Charlier Orthogonal Polynomial Sequence

In this talk, we derive the linear generating function for the Charlier Orthogonal Polynomial Sequence (OPS) by first principles and then by what we entitle the "Inverse Method." The Inverse Method transforms a three-term recurrence relation into a differential equation, with the solution being the desired generating function. We apply the Inverse Method to the Charlier OPS and then to a variation of the Laguerre polynomials, which shares a relationship with the Charlier Polynomials.

Mark Beckwith and Brad Spangler, Slippery Rock University, Armstrong 315
A Knot or Not a Knot?

A knot can be thought of as a piece of string without thickness such that, after desired manipulations are made, the ends of the string are connected. It is then best to think of a knot by examining the different projections of the knot in two dimensions; the Dowker notation is a particular way to describe a knot projection. We will examine some of the different properties of Dowker notation to determine which produce a knot.

7:55-8:10 PM

Tori Merten and Ben Cecchini, Washington & Jefferson College, Armstrong 119
Generating Functions: A Closer Look At The Fibonacci Numbers

Generating functions are a powerful tool which allow us to represent sequences in power series form. We used generating functions to determine a closed form for both the Fibonacci numbers and the binomial coefficients. We also produced an expression which generates Pascal's triangle. In this presentation, we detail these techniques and derivations.

Joey Cortez and Zack Servetnick, Slippery Rock University, Armstrong 206
The Alexander Polynomial Using Group Theory

The Fox algorithm, from group theory, offers a way to compute a knots Alexander Polynomial, an invariant. We will demonstrate a procedure for labeling a knot diagram to find its group presentation. Employing the Fox algorithm we will compute the Alexander Polynomial of a knot. Additionally, we will show that this method agrees with a combinatorial method for determining Alexander Polynomials.

Andre Schrock, University of Pittsburgh, Armstrong 303
The 3n + 1 Problem

A study of the "3n+1" problem through the use of residue classes illustrates a bridge between computer science and mathematical theory. The Collatz conjecture is a long standing mathematical idea, and has been verified up to very large numbers. This study's approach takes the idea of doing infinitely many calculations simultaneously and implements it through the use of residue classes.

Andrew Brown and Amanda Goodrick, Slippery Rock University, Armstrong 306
All Tangled Up? Unknot Me, Conway!

Conway notation and the unknotting number are two invariants we use to compare knots. We explore Conway notation looking for a connection between these two invariants with a focus on algebraic knots. The Conway notation will provide the tangles that build each knot. We look at the unknotting numbers of each tangle to see if there is an algebraic method to obtain the unknotting number of the original knot.

Greg Clark, Westminster College, Armstrong 315
Famous Sequences and Euclidean Algorithm Step Sizes

We will prove that the maximum step size for the Euclidean Algorithm is achieved using Fibonacci numbers and Lucas numbers of odd index. in particular, we will use a formula that provides an upper bound on the number of steps needed when using the Euclidean Algorithm on two natural numbers a and b. Furthermore, we will show that the upper bound is achieved for certain values of b.

8:15-8:30 PM

Rex W. Edmonds, Slippery Rock University, Armstrong 119
What is so Magical About Polygons?

Magic squares are 3×3 arrays of numbers for which the entries in each row and column sum to the same number, the magic sum. In this talk, we generalize this idea to discuss magic polygons; specifically, we will look at polygons with an even number of sides.

Maxwell Nurnberger, University of Pittsburgh, Armstrong 206
Synchronization with Flexible Frequencies

Synchrony appears in all walks of life from fireflys to humans to neural oscilators. Models of these events have been widely studied in the field of mathematics, and such physical events have been observed. I will present such a mathematical model that we have created and discuss some of its properites including coupled oscillators with adjustable frequencies.

Janessa Allshouse and Kaila Kramer, Slippery Rock University, Armstrong 303
The Tricolorability of Composite Knots

This talk explores the topics of knot composition and tricolorability. After definitions and several examples to illustrate these ideas, we will look at the relationship between these two topics. We will then move on to other invariants such as non-tricolorable knots, non-alternating knots, and p-colorability, showing how each of these can transfer to the composition of two or more knots.

Sarah Algee, Wheeling Jesuit University, Armstrong 306
Visual Representations of the Intersections of Sets

The Venn diagram, a picture of two or three interlocking circles, is a very important tool to show relationships in logic, statistics, and other sciences. However, if more than three parts to an argument are present, then it is impossible to use a Venn diagram to represent the data. Because many fields need to show relationships between more than three objects, it is useful to explore the use of other shapes to pictorially represent intersection of sets. The focus of this research was specifically on using four or more ellipses, rectangles, and triangles, as well attempting to establish guideline rules of congruence and if the shapes need to be equilateral.

Tanya Riston, Penn State University - Erie, Armstrong 315
Finite Distributive Lattices

In the early 1890s, Richard Dedekind was working on a revision of Dirichlet's *Vorlesungen über Zahlentheorie*, and asked the following question: Given three subgroups A, B, C of an abelian group G, how many different subgroups can you get by taking intersections and sums, e.g., $A + B$, $(A + B)\ C$, etc. The answer is 28. In looking at this and related questions, Dedekind was led to develop the basic theory of lattices, which he called Dualgruppen. Dedekind found that, "There is nothing new under the sun." Lattices, especially distributive lattices and Boolean algebras, arise naturally in logic, and thus some of the elementary theory of lattices had been worked out previously by Ernst Schröder in his book *Die Algebra der Logik*. Nonetheless, it was the connection between modern algebra and lattice theory, recognized by Dedekind, which was the driving force for the development of lattice theory as a subject, and is a primary interest today. Unfortunately, not much happened in lattice theory over the next thirty years. It was not until the 1930's when the development of universal algebra by Garrett Birkhoff, Oystein Ore and others, was Dedekind's work on lattices rediscovered. From then on, lattice theory has been an active and growing subject, in terms of its application to algebra and several areas of mathematics. In this talk, we will give an overview of finite distributive lattices and their properties. In addition we will give a few examples and current open questions of consideration.

8:35-8:50 PM

Meaghan Volek, Allegheny College, Armstrong 119
Mathematical March Madness

This presentation will investigate the least-squares method of approximation derived through the concept of projections. These mathematical concepts will then be adapted to the real world application of sport rankings. The Bowl Championship Series uses a similar method of approximation to produce the Top 25 College Football Rankings. This presentation will look into the least-squares method of approximation as a ranking system for college basketball. The ranking system produced can then be used to predict bracket style tournaments, including the NCAA Basketball Championships.

Kevin Meigh, Wheeling Jesuit University, Armstrong 206
Power Plant Problem: Fermat Point

My problem is called the power company problem. Suppose that a power company wants to select the most efficient location to build a power plant to supply cities A, B, and C. They want to build it in a spot where the sum of the distances from cities A, B, and C to the power plant is a minimum. Fermat and Torcelli have solved the power plant problem for three cities, and that solution is called the Fermat Point. The Fermat Point is not always in the interior of the triangle formed by the cities. If an angle of the triangle is greater than 120 degrees, the Fermat Point is at the vertex of that angle. Since Fermat and Torcelli have solved the power plant problem for three cities, I will solve it for greater than three cities and build off of their work. I can find the solution to more than three points through coordinate geometry and differential calculus. I will also try to find an algorithm to calculate the Fermat Point for more than three cities. I will try to geometrically prove that my Fermat Points generate the minimum possible sum of distances to the power plant.

Richard Ligo, Westminster College, Armstrong 303
Diagonal Diagrams: An Interesting Representation for a Specific Class of Links

Every link can be represented mathematically. One method for representing links is through a grid diagram. Every link has a corresponding grid diagram, and every grid diagram represents exactly one link. We define a special type of grid diagram called a diagonal diagram. From this definition, we prove that specific classes of diagonal diagrams represent the unknot, a torus knot, or multiple linked unknots. We also determine specific characteristics about the link represented by a diagonal diagram.

Zane Bowser and Katie Sopczynski, Slippery Rock University, Armstrong 306
Dowker Notations of Knots

In the field of knot theory, Dowker notation is an easy way to describe a given projection of a knot. Our talk will explore several questions concerning Dowker notation including: Why is there always an even and an odd number at each crossing? Why is Dowker notation a practical way to describe a knot? Is it true that for an n-crossing knot in the standard projection, there are always $2n$ different notations?

Oyinda Owoeye, University of Pittsburgh, Armstrong 315
Temporal Coding and Inhibition

This talk will explore the possibility of transforming a signal encoded in firing rate into a signal encoded in more precise spike timing.

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Steven Reich, University of Pittsburgh, Armstrong 119
The Impact of Short-term Synaptic Depression on Neural Variability

Neuronal activity in the brain is highly variable. An understanding of neural coding and dynamics requires a description of how this variability is generated and propagated in neuronal networks. Neurons communicate through the release of neurotransmitter vesicles across synaptic connections. Depletion of neurotransmitter vesicles induces a form of plasticity known as short term synaptic depression. We analyze a continuous time Markov chain model of a depressing synapse to investigate how short term depression affects the transfer of variability from one neuron to another.

Jennifer Murphy, California University of Pennsylvania, Armstrong 206
1 2 3 + A B C; Writing in the Mathematics Classroom

Utilizing writing exercises within the mathematics classroom can be beneficial to both students and instructors by enhancing topic comprehension, encouraging students to be more reflective about their own learning, and providing feedback for the instructor at both the high school and college level. I will discuss current research and theory, practical uses in mathematics, and strategies to implement writing effectively in the mathematics classroom.

Gabe Kramer, Penn State University - Erie, Armstrong 303
Vertex Replacement Rules

A vertex replacement rule replaces vertices of one graph with copies of another. This talk will discuss: examples of vertex replacements, what happens when these graphs are subject to an infinite number of replacements, the conditions required for vertices to be replaceable, and two theorems involving the isometry of graphs.

Adam Baumgardner and Tom Kryzwick, Slippery Rock University, Armstrong 306
Dowker Notation of Star Knots

Dowker notation is a way to classify knots. A star knot is one whose standard projection is the image of a (geometric) star. This talk will explore the Dowker notation of such. We will introduce the pattern found in applying this notation and present a new conjecture.

Ryan Slean and Steve Ways, Slippery Rock University, Armstrong 315
To p-Colorability and Beyond

A common invariant utilized in knot theory is the concept of p-colorability for a prime number p. Some knots may even have multiple primes, say p and q, such that the knot is both p-colorable and q-colorable. The question that remains is: if a knot is p-colorable and q-colorable, is the knot also (pq)-colorable? We will examine this question and also explore whether generalizations exist for p-colorability and alternating knots.

9:15-9:30 PM

Sam Taylor, Allegheny College, Armstrong 119
Continued Fractions and Their Applications

A continued fraction is an expression of the form

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\cdots + \cfrac{1}{a_n}}}}$$

where $a_0, a_1, \ldots, a_n$ are real numbers. Numbers of this form have their foundation in number theory, but have also been seen in many unique areas of mathematics throughout history. This presentation will cover some basic properties of continued fractions, such as converting rational numbers into continued fractions, as well as approximating irrational numbers, such as π, with continued fractions of this form. Some applications will also be given, including how the golden ratio and planetary motion can be approximated using continued fractions.

Brett Brown, Jimmy Valentino, and Brad Windhorst, Slippery Rock University, Armstrong 206
Matrix Unknotted: p-Coloring

In mathematics, a knot can be thought of as a closed string in space. One invariant used to classify knots is p-colorability. Determining which knots admit p-colorings for different primes, p, is complicated. By assigning variables to each strand of the knot, and then creating linear equations, we can get a matrix. From this matrix, we can demonstrate the p-colorability of the knot. This well-known method and many examples will be given.

Jackie Mitchell, Washington & Jefferson College, Armstrong 303
The Basel Problem

This paper explores the Basel Problem, which finds the exact value of the zeta function at 2. $\zeta(2)$ is the sum of the p-series with $p = 2$. We use two methods to prove that $\zeta(2) = \frac{\pi^2}{6}$. Furthermore, we explore two applications that involve the value $\zeta(2)$. One application shows how $\zeta(2)$ shows up in computing the probability that any two random positive integers are relatively prime. We also examine another version of the Gabriel horn, the Gabriel Wedding cake, which is a solid with finite volume but infinite surface area. It turns out that this finite volume can also be expressed in terms of $\zeta(2)$.

Paul Millington, University of Pittsburgh, Armstrong 306
Synchronization of Gamma Oscillations with Mixture of Slow and Fast Excitatory Synapses

Gamma rhythms (30-100 Hz range oscillations) are thought to be important in sensory perception. One mechanism that forms these rhythms is feedback between excitatory and inhibitory neurons. Excitatory synapses onto the inhibitory cells have fast and slow components. We explore the sensitivity and synchronization of rhythms as the ratio of two of these components varies. We use numerical bifurcation techniques to find the best ratios for maintaining a coherent rhythm over a range of input drives.